

# Algebraic Algorithm for Scheduling Data Retrieval in Multi-channel Wireless Data Broadcast Environments\*

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**Abstract.** Due to more and more customers keen on mobile services, we may face the mobile network congestion problem. Therefore, it is necessary to develop new data retrieval method to provide users with reliable and timely access to the data scourers. In this paper, we study the scheduling problem for retrieving data from multi-channel data broadcast environments. In general conditions, the most important two issues for queries in mobile computing systems are the energy cost and the query response time. In order to improve the query efficiency, we develop a randomized algebraic algorithm that takes both energy cost and access time into consideration to schedule the data retrieval process in multi-channel environments. It can be used in almost any broadcast environment, in which the data access frequencies, data sizes, and channel bandwidths can all be non-uniform.

**Keywords:** Wireless data broadcast, Multi-channel, Mobile computing, Data retrieval optimization.

## 1 Introduction

Wireless data broadcast is very suitable for disseminating public information to large number of mobile users. Generally, there are two major measures when evaluating the query efficiency in such environments. One is *access time* and the other is *energy efficiency*. The *access time* denotes the time interval between the moment a query starts to the moment all the requested data have been downloaded. Obviously, users prefer short *access time*. In addition, the power

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\* This work is supported in part by National Science Foundation under grant CNS1016320 and CCF0829993.

supply is finite for mobile devices, which means *energy efficiency* is also very important when design data retrieval method in mobile computing environments.

In a general data broadcast network, a base station disseminates data to mobile users via one or multiple channels; so there are totally two entities: *mobile client* and *base station*. Fig. 1 shows a typical broadcast network. A base station can send information via radio waves to large number of mobile clients simultaneously, and the cost at the server site will not change as the number of clients increases. For instance, the base station  $B_2$  in Fig. 1 provide services to more users, but its costs is nearly the same as that of  $B_1$ . Moreover, the *mobile client* may has two modes: *active mode* and *sleep mode*. With the help of index, clients can get the arriving time of their requested data in advance and “sleep” to save energy when there are no data of interests.

Although, it is relatively easy for data retrieving if all the data are scheduled on one channel, users may prefer partition the data onto multiple channels to reduce the average expected access time. But it should be noticed that if a client is downloading a data from channel  $c_i$  at time  $t_0$ , then it cannot switch to channel  $c_j$ , where  $j \neq i$ , to download another data at time  $t_0 + 1$ . The reason is that switching the channels takes time, and if a client want to download data from another channel, it needs one time slot for channel switching. Fig. 2 gives a typical process of data retrieval in multi-channel broadcast environments. The query data set is  $\{d_1, d_3, d_5\}$ , and a user can download data object  $d_1$  and  $d_3$  from channel  $c_1$ , and then switch to channel  $c_3$  at time 6 to download data object  $d_5$  at time 7. However, after time 5, the user cannot switch from channel  $c_1$  to  $c_2$  to download data  $d_5$  at time 6. From Fig. 2, we also can get that the bandwidths of different channels are non-necessarily the same. Actually, the bandwidth of channel  $c_2$  is twice as that of  $c_1$  or  $c_3$ , thus  $d_3$  or  $d_5$ , which take two time slots on  $c_1$  or  $c_3$ , can be broadcasted in one time slot by  $c_2$ .

In most situations, it is much more likely a client query a set of data instead of only one data at a time. After obtaining the locations of requested data items, we need to make a schedule to download the data one by one in some order.

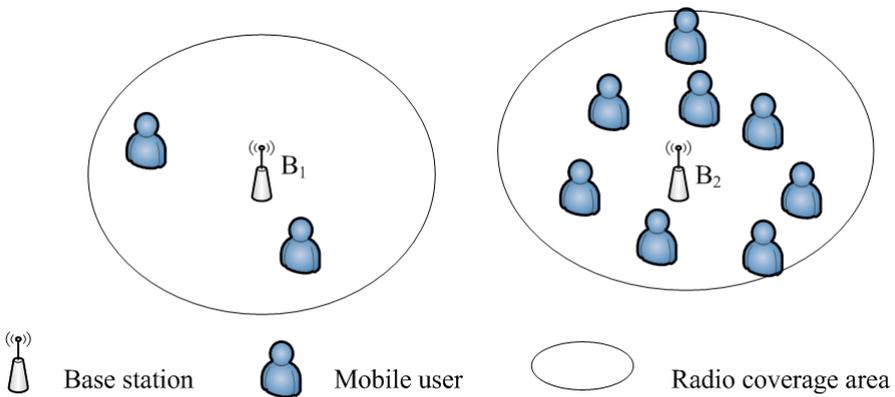


Fig. 1. Data Broadcast Network

An unwise retrieving schedule may result in long access time and unnecessary energy consumption. Usually, the energy consumption is evaluated base on the following two metrics: 1) tune-in time and 2) the number of channel switchings. Assume the arriving time of each requested data item is already known from the index, then the energy consumption depends purely on the number of channel switching happens during the retrieval process. In this paper we propose a randomized algebraic algorithm to reduce the access time and channel switchings for data retrieval in multi-channel environments. It can be used in almost any data broadcast programs, in which the data access frequencies, data sizes, and channel bandwidths can all be non-uniform.

The remainder of this paper is organized as follows. Section 2 presents the related works to wireless data broadcast. In section 3, we give an algebraic algorithm that considers both access latency and energy cost to get optimal solutions for data retrieving in multi-channel environments. Finally, in section 4, we conclude this paper.

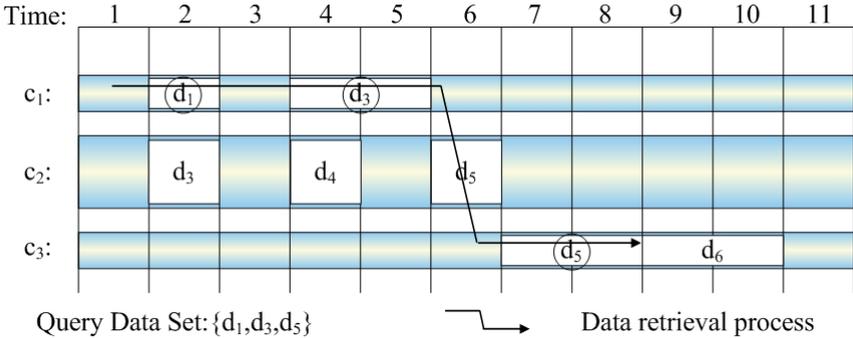


Fig. 2. Data Retrieval Process

## 2 Previous Works

Scheduling is an important issue in the area of wireless data broadcast. Acharya et al. first proposed the scheduling problem for data broadcast [1], and Prabhakara et al. suggested the multi-channel model for data broadcast to improve the data delivery performance [2]. After that, many works have been done for scheduling data on multiple channels to reduce the expected access time [5,6,12]. Besides, some researches began to study how to allocate dependent data on broadcast channels. (see e.g. [14,10,11]). With respect to index, many methods have been proposed to improve the search efficiency in data broadcast systems (see e.g. [8,3,9,10,11]). Furthermore, Jung et al. proposed a tree-structured index algorithm that allocates indices and data on different channels [7]. Lo and Chen designed a parameterized schema for allocating indices and data optimally on multiple channels such that the average expected access latency is minimized [4].

In terms of data retrieval scheduling, Hurson et al. proposed two heuristic algorithms for downloading multiple data items from multiple channels [15]. Shi

et al. investigated how to schedule multiple processes to download a set of data items [16]. Both of them investigate the data retrieval problem by assuming that the data are allocated on multiple channels without replication. However, as shown in the prior studies [1,20,21,22], employing data replication in data broadcast programs will reduce the expected access time. Fig. 3 shows why disseminating replicative data by multiple channels can reduce both access time and energy consumption. The first program allocates data without replication.  $d_1$  and  $d_2$  are separately scheduled on channels  $c_1$  and  $c_2$ . we can download  $d_1$  or  $d_2$  in one time slot, but we need at least 3 time slots and 1 channel switching to download both  $d_1$  and  $d_2$  in such a system. If allocating data on channels like the way of program 2, we can still retrieve each datum in one time slot and we can retrieve both of them in 2 time slots without channel switching.

In this paper, we develop an algebraic algorithm for solving the problem of retrieving multiple data from multiple channels, in which the data can be non-uniform length and are replication-allowed to be broadcasted via multiple channels.

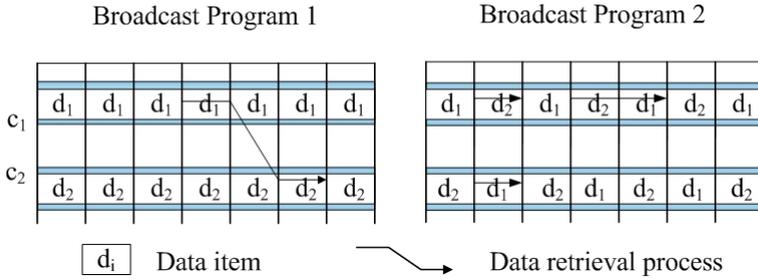


Fig. 3. Two Types of Broadcasting Database

### 3 Methodology

In this section we present an algebraic algorithm for the data retrieval problem in multi-channel environments. It can detect if a given problem has a schedule to download all the requested data before time  $t$  and with at most  $h$  channel switchings in  $O(2^k(hnt)^{O(1)})$  time, where  $n$  is the number of channels and  $k$  is the number of required data items. We first give a problem definition in section 3.1, and then present the algorithm in section 3.2.

#### 3.1 Problem Description

Let's consider a mobile user wants to query a set of data  $D = \{d_1, d_2, \dots, d_k\}$ , which are broadcasted via channels in  $C = \{c_1, c_2, \dots, c_n\}$ . We assume the locations of all the data in  $D$  are known; and we also assume each channel is partitioned into discrete time slots and one time slot is the smallest unit for storing data. Let a tuple  $s = \{i_s, j_s, t_s, t'_s\}$  denote the data  $d_{i_s}$  can be downloaded from channel  $c_{j_s}$  during the time span  $[t_s, t'_s]$ , then it is clear that a valid

data retrieval schedule is a sequence of  $k$  intervals  $s_1, s_2, \dots, s_k$ , each tuple corresponds to a distinct data item in  $D$  and there is no conflicts between any two of the  $k$  tuples.

**A Decision Problem:** Given a data set  $D$ , a channel set  $C$ , a time threshold  $t$  and a switching threshold  $h$ , find a valid data retrieval schedule to download all the data in  $D$  from  $C$  before time  $t$  with at most  $h$  switchings.

To solve the above decision problem, we developed a randomized algebraic algorithm. We present it in detail next.

### 3.2 Algorithm

The basic idea of our algebraic algorithm is that for each data item  $d_i \in D$ , where  $D$  is the query data set, we create a variable  $x_i$  to represent it. Therefore, given  $D = \{d_1, d_2, \dots, d_k\}$ , we construct a variable set  $X = \{x_1, x_2, \dots, x_k\}$ . We then design a circuit  $H_{t,h,n}$  such that a schedule without conflict will be generated by a multilinear monomial in the sum of product expansion of the circuit. A multilinear monomial is a monomial such that each variable has degree exactly one, for examples,  $x_3x_5x_6$  is a multilinear monomial, but  $x_3x_5^3x_6^2$  is not. The existence of schedules to download all the data items in  $D$  from the multiple channels of  $C$  is converted into the existence of multilinear monomials of  $H_{t,h,n}$ . Replace each variable by a specified binary vector can remove all of the non-multilinear monomials by converting them to zero. Thus, the data retrieval problem is transformed into testing if a multivariate polynomial is zero. It is well known that randomized algorithms can be used to check if a circuit is identical to zero in polynomial time.

**Lemma 1.** *There is a polynomial time algorithm such that given a channel  $c_i$ , a time interval  $[t_1, t_2]$ , and an integer  $m$ , it constructs a circuit of polynomial  $P_{i,t_1,t_2,m}$  such that for any subset  $D' = \{d_{i_1}, \dots, d_{i_m}\} \subseteq D$  which has a size of  $m$  and is downloadable in the time interval  $[t_1, t_2]$  from channel  $c_i$ , the product expansion of  $P_{i,t_1,t_2,m}$  contains a multilinear monomial  $x_{i_1}x_{i_2} \dots x_{i_m}$ .*

*Proof.* We can use a recursive way to compute the circuit  $P_{i,t_1,t_2,m}$  in polynomial time.

1.  $P_{i,t_1,t_2,0} = 0$ .
2.  $P_{i,t_1,t_2,1} = \sum_j x_j$ ,  $x_j \subseteq X$  and the corresponding data  $d_j$  is entirely in the time interval  $[t_1, t_2]$  of channel  $c_i$ .
3.  $P_{i,t_1,t_2,l+1} = \sum_j x_j \cdot P_{i,t_1,t'_2,l} + P_{i,t_1,t'_2,l+1}$ ,  $d_j$  starts at time  $t'_2 + 1$  and ends before time  $t_2$  on channel  $c_i$ .

When computing  $P_{i,t_1,t_2,l+1}$ ,  $x_j$  multiplies  $P_{i,t_1,t'_2,l}$  is based on the case that  $d_j$  is downloadable from time  $t'_2 + 1$  to  $t_2$  in the final phase, and the other  $l$  data items are downloadable before time  $t'_2$ . The term  $P_{i,t_1,t'_2,l+1}$  is the case that  $l + 1$  items are downloaded before time  $t'_2$ . Note that the parameter  $m$  in  $P_{i,t_1,t_2,m}$  controls the total number of data to be downloaded.

**Definition 1.** A subset data items  $D' = \{d_{i_1}, \dots, d_{i_m}\} \subseteq D$  is  $(i, t, h)$ -downloadable if we can download all data items in  $D'$  before time  $t$ , the total number of channel switches is at most  $h$ , and the last downloaded item is from channel  $c_i$ .

**Lemma 2.** Given two integers  $t$  and  $h$ , there is a polynomial time algorithm to construct a circuit of polynomial  $F_{i,t,h,m}$  such that for any  $(i, t, h)$ -downloadable subset  $D' = \{d_{i_1}, \dots, d_{i_m}\} \subseteq D$ , the product expansion of  $F_{i,t,h,m}$  contains a multilinear monomial  $(x_{i_1}, \dots, x_{i_m})Y$ , where  $Y$  is a multilinear monomial doesn't include any variable in  $X$ .

*Proof.* We still use a recursive way to construct the circuit. Some additional variables are used as needed. Without loss of generality, we assume the data retrieval process start at time 0.

1.  $F_{i,t,0,0} = 0$ .
2.  $F_{i,t,0,1} = P_{i,1,t,1} \cdot y_{i,t,0,1}$ .
3.  $F_{i,t,h'+1,m'+1} = y_{i,t,h'+1,m'+1,0}(\sum_{t' < t} F_{i,t',h'+1,m'} \cdot P_{i,t'+1,t,1})$   
 $+ y_{i,t,h'+1,m'+1,1}(\sum_{j \neq i} \sum_{t' < t} F_{i,t'-1,h',m'} \cdot P_{i,t'+1,t,1})$

The computing of  $F_{i,t,h'+1,m'+1}$  is based on two cases, and we use two variables,  $y_{i,t,h'+1,m'+1,0}$  and  $y_{i,t,h'+1,m'+1,1}$ , to mark them respectively. We now present an algorithm that involves one layer randomization to determine if there is a schedule to download all the data items in  $D$  before time  $t$  and with at most  $h$  channel switchings.

**Theorem 1.** There is an  $O(2^k(hnt)^{O(1)})$  time randomized algorithm to determine if there is a scheduling to download  $k = |D|$  data items before time  $t$  and the number of channel switches is at most  $h$ , where  $n$  is the total number of channels.

*Proof.* By Lemma 2, we can construct a circuit  $H_{t,h,n} = \sum_{i=1}^n F_{i,t,h,k}$  in polynomial time. It is easy to see there is a scheduling for downloading the  $k$  data items before time  $t$  and with  $h$  channel switches, if and only if the sum product expansion of  $H_{t,h,n}$  has a multilinear monomial  $(x_1, \dots, x_k)Y$ .

Replace each  $x_i$  by a vector  $w_i = w_0^T + v_i^T$ , where  $w_0$  is the all-zeros vector of dimension  $k$ , and  $v_i$  is a binary vector of dimension  $k$  with its  $i^{th}$  element is 1 and all other elements are 0. Assume  $k = 3$ , we define the following operations:

$$v_a \cdot v_b = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} (a_1 + b_1)(\text{mod}2) \\ (a_1 + b_2)(\text{mod}2) \\ (a_1 + b_3)(\text{mod}2) \end{pmatrix} \quad (1)$$

$$(v_a + v_b) \cdot v_c = v_a \cdot v_c + v_b \cdot v_c \quad (2)$$

By Equation 1 and 2, for any  $k$ -dimensional binary vector  $w' = w_0 + v$ , we have  $w'^2 = w_0^2 + 2w_0 \cdot v + v^2 = w_0 + 2(w_0 \cdot v) + w_0 = 2(w_0 \cdot v) + 2w_0 = 0$ ,

because of the coefficients are in the field of  $G_2$ . The replacement  $x_i = w_i (i = 1, \dots, m)$  make all the non-multilinear monomials become zero. Meanwhile, all the multilinear monomials remain non-zero. Hence, it is clear that there is a scheduling to download all the data items in  $D$  before time  $t$  and with at most  $h$  channel switchings if and only if  $H_{t,h,n|x_i=w_i(i=1,\dots,k)}$  is a non-zero polynomial. The variables in  $Y$  makes it impossible to have cancelation when adding two identical multilinear monomials, which can be generated from different paths with variables in  $\{x_1, \dots, x_k\}$ . It is well known that randomized algorithms can be used to check if a circuit is identical to zero in polynomial time [17], [18].

The algorithm generates less than  $2^k$  terms during the computing process since there are at most  $2^k$  distinct binary vectors. Therefore, the computational time is  $O(2^k(nht)^{O(1)})$ .

## 4 Conclusions

In this paper, we take both access time and channel switchings into consideration to investigate the minimum cost data retrieval problem in multi-channel data broadcast environments. The algorithm proposed can detect if a given data retrieval problem has a solution with access time  $t$  and number of switchings  $h$  in  $O(2^k(hnt)^{O(1)})$  time, where  $n$  is the number of channels and  $k$  is the number of requested data items. It can be used in almost any broadcast environment, in which the data access frequencies, data sizes, and channel bandwidths can all be non-uniform.

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