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Constructing weakly connected dominating set for secure clustering in distributed sensor network

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Abstract Secure clustering problem plays an important role in distributed sensor networks. *Weakly Connected Dominating Set* (WCDS) is used for solving this problem. Therefore, computing a minimum WCDS becomes an important topic of this research. In this paper, we compare the size of *Maximal Independent Set* (MIS) and minimum WCDS in unit disk graph. Our analysis shows that five is the least upper bound for this ratio. We also present a distributed algorithm to produce a weakly connected MIS within a factor 5 from the minimum WCDS.

Keywords DSN · Security · Weakly connected dominating set · Maximal independent set · Unit disk graph

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1 Introduction

Distributed sensor networks have been widely used in many military and public-oriented operations such as area monitoring, target tracking and moving detection. DSNs differ from the traditional wireless networks. They have a huge number of sensors and large coverage area. DSN is dynamic which means we can add or delete any sensors at any time. Secure clustering problem is well studied in sensor networks, but it might not be always true for DSNs.

Pathan and Hong (2009) proposed a novel algorithm called key-predistribution-based weakly connected dominating set to solve the secure clustering problem in DSNs. They consider the DSN as a *Unit Disk Graph* (UDG) in which all vertices lie in the Euclidean plane and an edge (u, v) exists iff the distance between u and v is at most one. In the undirected graph $G = (V, E)$, a *Dominating Set* (DS) is a subset $S \subseteq V$ such that for every vertex $v \in V$, either $v \in S$, or there exist an edge $(u, v) \in E$ for some $u \in S$. If the subgraph induced by DS is connected, we call it *Connected Dominating Set* (CDS). The subgraph weakly induced by W is S_w=$(N[W], E \cap (W \times N[W]))$, where $N[W]$ is a set of one-hop neighbor of W and W. The set W is a *Weakly Connected Dominating set* (WCDS) if S_w is connected and W is a DS. The size of WCDS is usually less number of nodes is needed for the connectivity than CDS (Pathan and Hong 2009). Our goal is to find the minimum size of WCDS. However, to find a minimum size of WCDS is NP-hard (Dunbar et al. 1997). Therefore, how to find an approximation algorithm becomes a new challenge. Chen and Liestman (2002) proposed an approximation algorithm with ratio $O(ln\Delta)$, where Δ is the maximum degree of the input network. Alzoubi et al. (2003) and his co-workers present two distributed algorithms with the ratio 5 and 122.5. Dubhashi et al. (2003) gave another approximation algorithm with the same ratio as Chen's (2003).

A *Maximal Independent Set* (MIS) is a subset $M \subseteq V$ such that for every pair of vertices in M are not adjacent, and no independent vertex can be add in to M. u and v are two independent vertices if $|uv| > 1$. Clearly, any MIS is a DS. It is a popular way to construct MIS for a DS (Wu et al. 2006; Alzoubi et al. 2003). In this paper, we use MIS to approximate the minimum WCDS. Therefore, the approximation ratio will depend on the comparison of the size of MIS and the minimum WCDS. It has been known that the ratio of sizes between MIS and the minimum WCDS is at most five (Alzoubi et al. 2003).

In this paper, we show that the five is the least upper bound for this ratio. We also present a distributed algorithm to produce a weakly connected MIS within a factor 5 from the minimum WCDS.

This paper is organized as follows. In Sect. 2, we give the lower bound result for the ratio of sizes between MIS and the minimum WCDS. We will present a distributed algorithm in Sect. 3. Simulation results are shown in Sect. 4. Finally, we conclude the paper in Sect. 5.

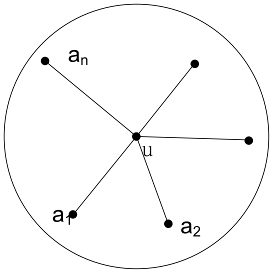


Fig. 1 Five independent points

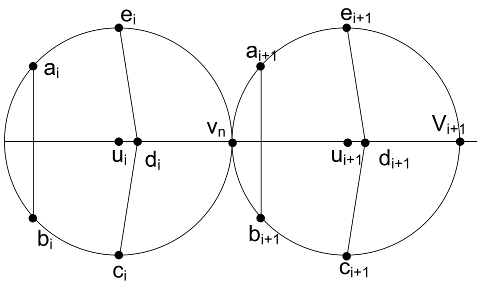


Fig. 2 An example for $|M| = 5|D|$

2 MIS and minimum WCDS

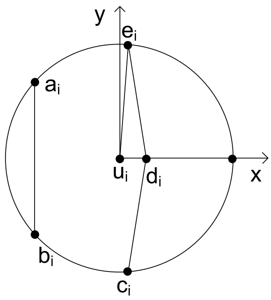
Throughout this paper, our study is on UDG. Therefore, a set of vertices is independent if and only if every pair of vertices in the set has distance more than one. In this section, we show the following.

Theorem 1 *The least upper bound for the ratio of sizes between MIS and the minimum WCDS for UDG is* 5.

Proof It is easy to show that 5 is an upper bound. In fact, consider an MIS M and a minimum WCDS D. For any vertex u, the disk with radius one and center u is called the *dominating disk* of u. Suppose $u \in D$ and the dominating disk of u contains n elements of M, $a_1, \dots, a_n$. Connect $ua_1, ua_2, \dots, ua_n$ and assume that they are in clockwise ordering. Since $a_1, a_2, \dots, a_n$ are independent, we have $|a_1a_2| > 1, \dots, |a_na_1| > 1$. It follows that $\angle a_1ua_2 > \pi/3, \dots, \angle a_nua_1 > \pi/3$ Fig. 1. Hence, $n \le 5$. Therefore, $|M| \le 5|D|$.

To see that 5 is the least upper bound, we show an example in which $|M| = 5|D|$ where M is an MIS and D is a minimum WCDS. First, construct a sequence of points $u_1, v_1, u_2, v_2, \dots, u_n, v_n$ on a line ℓ such that $|u_1v_1| = |v_1u_2| = |u_2v_2| = \cdots = |u_nv_n| = 1$. As shown in (Fig. 2), on the boundary of the dominating disk of each u_i, construct four points a_i, b_i, c_i, e_i such that segment a_ib_i is perpendicular to line ℓ and $\angle e_iu_ia_i = \angle a_iu_ib_i = \angle b_iu_ic_i = \frac{\pi}{3} + \varepsilon$ where $\varepsilon > 0$ is a small constant. Construct the fifth point d_i in dominating disk of u_i such that d_i is on line ℓ and at the middle between segments a_ib_i and $a_{i+1}b_{i+1}$. We study the UDG G with vertex set

Fig. 3 Calculation of $|d_i e_i|$



$\{a_i, b_i, c_i, d_i, e_i, u_i, v_i \mid i = 1, 2, \ldots, n\}$. Note that

$$|e_i a_i| = |a_i b_i| = |b_i c_i| = 2\sin\left(\frac{\pi}{6} + \frac{\varepsilon}{2}\right) > 1,$$

$$|d_i a_i| = |d_i b_i| = |d_i a_{i+1}| = |d_i b_{i+1}| = \sqrt{1 + \frac{|a_i b_i|^2}{4}} > \sqrt{1.25}.$$

Put a coordinate system with origin u_i as shown in Fig. 3. Then we have $\angle e_i u_i y = \frac{3\varepsilon}{2}$ and $|u_i d_i| = 1 - \cos(\frac{\pi}{6} + \frac{\varepsilon}{2})$. Thus, points d_i and e_i have coordinates $(1 - \cos(\frac{\pi}{6} + \frac{\varepsilon}{2}), 0)$ and $(\sin(\frac{3\varepsilon}{2}), \cos(\frac{3\varepsilon}{2}))$, respectively. Hence,

$$|d_i c_i| = |d_i e_i| = \sqrt{\left(\cos\left(\frac{3\varepsilon}{2}\right)\right)^2 + \left(1 - \cos\left(\frac{\pi}{6} + \frac{\varepsilon}{2}\right) - \sin\left(\frac{3\varepsilon}{2}\right)\right)^2}$$

$$\to \quad \sqrt{1 + \left(1 - \frac{\sqrt{3}}{2}\right)^2} \quad \text{as } \varepsilon \to 0,$$

we have $|d_i c_i| = |d_i e_i| > 1$ for sufficiently small $\varepsilon > 0$. Therefore, $M = \{a_i, b_i, c_i, d_i, e_i \mid i = 1, 2, \ldots, n\}$ is an independent set and clearly an MIS. It is easy to see that $D = \{u_1, u_2, \ldots, u_n\}$ is the minimum WCDS for G and we have $|M| = 5|D|$. □

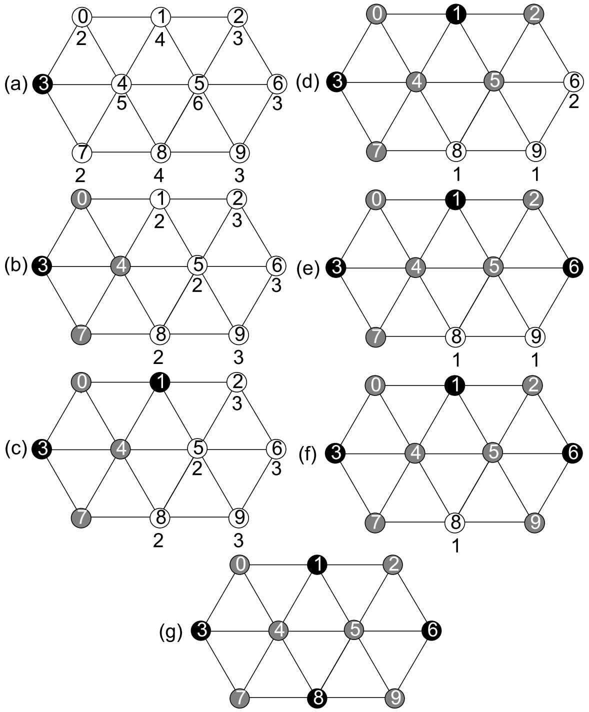
3 Distributed algorithm

We propose a distributed algorithm for constructing an MIS as follows.

Initial. Assign every vertex a positive integer as its ID; different vertices have different IDs. Color a vertex in black and all other vertices in white. Every white vertex sends a message "initial" to its neighbors. Based on received messages, every white vertex u compute a w-value $w(u)$, which is the number of white vertices in its neighbors. Then all white vertices go to *sleep*.

Step 1. Each black vertex sent message "black" to its neighbors. When a white vertex receive a message "black", it is activated, change its color to grey, and send out a message "grey" to its neighbors. Every black or grey vertices stop computing.

Fig. 4 Example for algorithm



Step 2. When a white vertex u received a message "grey", it is activated, update w-value $w(u) \leftarrow w(u) -$ (number of received "grey"-messages), and send $w(u)$ and its ID to its neighbors.

Step 3. Each activated white vertex u compares its w-value $w(u)$ with received w-values from its neighbors. If (a) $w(u)$ is larger than every received w-value or (b) $w(u)$ is the largest w-value and its ID is the smallest one among those neighbors from whom u received w-value equal to $w(u)$, then u changes it color to black.

Repeat Steps 1–3 until all vertices stop computing.

We show an example for this algorithm in Fig. 4. (a) Initial all the vertices, make vertex 3 black, all other vertices have their ID and the w-value below them. (b) Vertex 3 sends message "black" to vertices 0, 4, 7. They change their color to grey and send a message "grey" to their neighbors 1, 5, 8. When vertices 1, 5, 8 receives the "grey" message, they update their w-value. (c) Because vertices 1, 5, 8 have the same w-value, color vertex 1 black. Repeat steps 1–3 which shows in (d), (e), (f). Finally, all vertices stop computing, we get (g). The black vertices are MIS.

Note that this distributed algorithm produces a weakly-connected MIS. By Theorem 1, we have

Theorem 2 *Above distributed algorithm produces a weakly connected MIS within a factor of five from the minimum WCDS.*

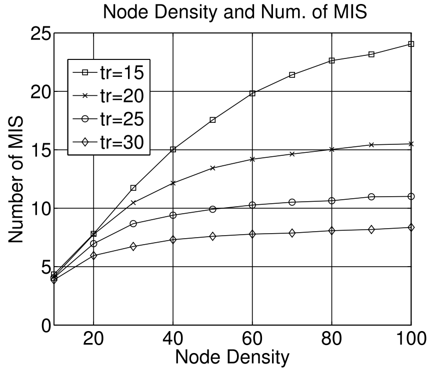


Fig. 5 Relationship between node density and number of MIS nodes

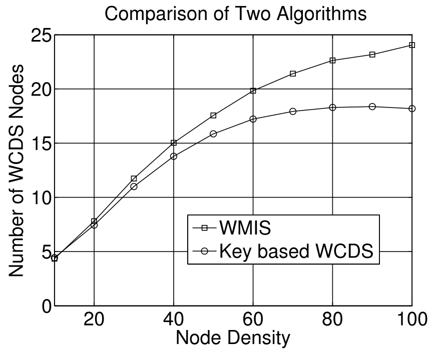


Fig. 6 Comparison of two algorithms

4 Simulation results

In this section, we evaluated the performance of the distributed algorithm proposed in Sect. 3 and the key based weakly connected dominating set algorithm presented by Pathan and Hong (2009). The simulator is implemented in Java language to evaluate the two algorithms. In our simulation, we generated random graphs with 10 to 100 nodes deployed randomly in a fixed area of 100 m $\times$ 100 m, and all nodes have the same transmission range. Figure 5 shows the performance of our algorithm. We varied the transmission range among 15 m, 20 m, 25 m, 30 m. The results show that the number of MIS nodes decrease when we increase the transmission range. Figure 6 shows the comparison of the two algorithms. Although Pathan and Hong (2009) has a better result, their algorithm is heuristic. Heuristic algorithms always have a better simulation result than approximation algorithms. Figure 6 shows that the difference become bigger when the nodes become dense.

5 Conclusion

In this paper, we studied minimum weakly connected dominating set problem. We showed that 5 is the least upper bound for the ratio of sizes between an MIS and the minimum WCDS. We also designed a distributed algorithm to produce a weakly connected MIS and hence it reaches the least performance ratio 5 for the minimum WCDS among those approximations using MIS.

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