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Maximum lifetime connected coverage with two active-phase sensors

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Abstract A sensor with two active phrases means that active mode has two phases, the full-active phase and the semi-active phase, which require different energy consumptions. A full-active sensor can sense data packets, transmit, receive, and relay the data packets. A semi-active sensor cannot sense data packets, but it can transmit, receive, and relay data packets. Given a set of targets and a set of sensors with two active phrases, find a sleep/active schedule of sensors to maximize the time period during which active sensors form a connected coverage set. In this paper, this problem is showed to have polynomial-time $(7.875 + \varepsilon)$-approximations for any $\varepsilon > 0$ when all targets and sensors lie in the Euclidean plane and all sensors have the same sensing radius R_s and the same communication radius R_c with $R_c \geq 2R_s$.

Keywords Maximum lifetime · Coverage · Wireless sensor network

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1 Introduction

There are many optimization problems on energy efficiency in wireless sensor networks [3,5,10–13]. In this paper, we study one of them.

Consider a wireless sensor network consisting of sensors with active mode and sleep mode where active mode has two phases, the full-active phase and the semi-active phase. A full-active sensor can sense data packets, transmit, receive, and relay the data packets. A semi-active sensor cannot sense data packets, but it can transmit, receive, and relay data packets. Clearly, a sensor in full-active phase consumes more energy than in semi-active phase. This model has been studied in the literature [15].

In many cases, sensors are randomly deployed into hostile environment, such as battlefield, inaccessible area with chemical or nuclear pollution, so that recharging batteries of sensors is a mission impossible. This means that the lifetime of each sensor depends on energy consumption. For simplicity, we may assume the battery of each sensor contains a unit amount of energy.

In this paper, we study the problem of environment monitoring, i.e., given a set of targets, sensors are used to sense them and to delivery obtained data to data center. For this purpose, the wireless sensor network is said to be alive if it can do our job, i.e., it satisfied the following two conditions:

(A1) Every target is monitored by a full-active sensor, and
(A2) all active sensors induce a connected subgraph.

We are interested in studying the problem of active/sleep scheduling of sensors to maximize the lifetime of given wireless sensor network.

This problem is called the *maximum lifetime connected coverage*, which is equivalent to a min-max problem as follows: Suppose the lifetime of wireless network is fixed and the energy of every sensor is unlimited. Find an active/sleep schedule to minimize the maximum energy consumption of a sensor.

In this paper, we show that the maximum lifetime connected coverage problem has polynomial-time $(7.875+\varepsilon)$-approximations for any $\varepsilon > 0$ when all targets and sensors lie in the Euclidean plane and all sensors have the same sensing radius R_s and the same communication radius R_c with $R_c \geq 2R_s$. So does the min-max problem.

2 A primal-dual method

Let S be the set of all sensors. Assume all sensor are of same type. Let u be the energy consumption of a full-active sensor during a unit time period and v the energy consumption of a semi-active sensor during a unit time period. Assume $u \geq v$. A set pair p is called an *active sensor set pair* if $p = (p_1, p_2)$ where p_1 is a set of full-active sensors and p_2 is a set of semi-active sensors with $p_1 \cap p_2 = \emptyset$. For any active sensor set pair p, define

$$a_{s,p} = \begin{cases} u & \text{if } s \in p_1, \\ v & \text{if } s \in p_2, \\ 0 & \text{otherwise.} \end{cases}$$

Let $\mathcal{C}$ be the collection of all active sensor set pairs satisfying conditions (A1) and (A2). The maximum lifetime connected coverage problem can be formulated as the following linear programming:

$$\begin{aligned} \max \quad & \sum_{p\in\mathcal{C}} x_p \\ \text{subject to} \quad & \sum_{p\in\mathcal{C}} a_{s,p} x_p \le 1 \quad \text{for } s \in S \\ & x_p \ge 0 \quad \text{for } p \in \mathcal{C}. \end{aligned}$$

Its dual is as follows.

$$\begin{aligned} \min \quad & \sum_{s\in S} y_s \\ \text{subject to} \quad & \sum_{s\in S} a_{s,p} y_s \ge 1 \quad \text{for } p \in \mathcal{C}, \\ & y_s \ge 0 \quad \text{for } s \in S. \end{aligned}$$

Motivated from the work of Garg and Könemann [7], we design the following primal-dual algorithm.

Initially, choose $x_p = 0$ for all $p \in \mathcal{C}$ and $y_s = \delta$ for all $s \in S$ where δ is a positive constant which will be determined later.

In each iteration, we first compute a ρ-approximation solution p^* for

$$\min_{p\in\mathcal{C}} \sum_{s\in S} a_{s,p} y_s. \tag{1}$$

and then compute a solution s^* for

$$\max_{s\in S} a_{s,p^*}.$$

Next, update x_p and y_s as follows:

(B1) x_p does not change for $p \neq p^*$, and

$$x_{p^*} \leftarrow x_{p^*} + \frac{1}{a_{s^*,p^*}}.$$

(B2) y_s does not change for $s \notin p_1^* \cup p_2^*$, and

$$y_s \leftarrow y_s \left(1 + \theta \frac{a_{s,p^*}}{a_{s^*,p^*}}\right)$$

for $s \in p_1^* \cup p_2^*$ where θ is a constant chosen later.

Clearly, after each iteration, some values of y_s are increased so that some constraints in dual linear programming get satisfied or close to satisfied. The algorithm will stop when $(y_s, s \in S)$ becomes dual feasible, that is, all constrains in dual linear programming are satisfied.

There are two important properties at end of algorithm.

Lemma 1 *At end of algorithm, $(x_p, p \in \mathcal{C})$ may not be primal feasible. However, $(x_p/\tau, p \in \mathcal{C})$ is a primal-feasible solution for $\tau = \frac{(v/u)\ln\frac{1+\theta}{v\delta}}{\ln(1+\theta v/u)}$.*

Proof First, note that at end of the algorithm, $y_s < (1+\theta)/v$. In fact, when y_s gets updated, following facts must hold:

(a) $(y_s, s \in S)$ is not dual-feasible.
(b) $\sum_{s \in S} a_{s,p^*} y_s < 1$. ((b) follows from (a).)
(c) $s \in p_1^* \cup p_2^*$.

It follows from (b), (c) that $y_s < 1/v$ before y_s receives any value change. After y_s is updated, we have

$$y_s < \left(1 + \theta \frac{a_{s,p^*}}{a_{s^*,p^*}}\right) / v \leq (1+\theta)/v.$$

Therefore, at end of algorithm, $y_s < (1+\theta)/v$.

Now, consider a constraints in the primal linear programming,

$$\sum_{p \in \mathcal{C}} a_{s,p} x_p \leq 1,$$

which may not be satisfied after x_p is updated. If updating x_p gives the value of $\sum_{p \in \mathcal{C}} a_{s,p} x_p$ an increase in $\frac{a_{s,p^*}}{a_{s^*,p^*}}$, then the value of y_s is increased by multiplying a factor $1 + \theta \frac{a_{s,p^*}}{a_{s^*,p^*}}$. Note that $\frac{a_{s,p^*}}{a_{s^*,p^*}}$ has only two nonzero values, v/u and 1. Suppose $\frac{a_{s,p^*}}{a_{s^*,p^*}}$ takes value v/u for k times and 1 for ℓ times. Then the value of $\sum_{p \in \mathcal{C}} a_{s,p} x_p$ receives an increase in $k(v/u) + \ell$ and

$$(1 + \theta v/u)^k (1+\theta)^\ell \leq \frac{1+\theta}{v\delta}$$

since initially $y_s = \delta$. Moreover, initially, $\sum_{p \in \mathcal{C}} a_{s,p} x_p = 0$. Thus, at end of algorithm, the value of $\sum_{p \in \mathcal{C}} a_{s,p} x_p$ is $k(v/u) + \ell$, which is upper-bounded by the maximum value of the following linear programming with respect to k and ℓ:

$$\begin{aligned} \max \quad & k(v/u) + \ell \\ \text{subject to} \quad & k \ln(1 + \theta v/u) + \ell \ln(1+\theta) \leq \ln \frac{1+\theta}{v\delta} \\ & k \geq 0, \ell \geq 0. \end{aligned}$$

Note that in any linear programming, the maximum value of objective function is reached at an extreme point. The feasible domain of above linear programming has three extreme points

$$(0,0), \quad \left(0, \frac{\ln \frac{1+\theta}{v\delta}}{\ln(1+\theta)}\right), \quad \left(\frac{\ln \frac{1+\theta}{v\delta}}{\ln(1+\theta v/u)}, 0\right),$$

which give objective function values

$$0, \quad \frac{\ln \frac{1+\theta}{v\delta}}{\ln(1+\theta)}, \quad \frac{v}{u} \cdot \frac{\ln \frac{1+\theta}{v\delta}}{\ln(1+\theta v/u)},$$

respectively. Since $\frac{z}{\ln(1+\theta z)}$ is strictly monotone decreasing for $z \leq 1$, we have

$$0 < \frac{\ln \frac{1+\theta}{v\delta}}{\ln(1+\theta)} < \frac{v}{u} \cdot \frac{\ln \frac{1+\theta}{v\delta}}{\ln(1+\theta v/u)}.$$

Therefore, at end of algorithm,

$$\sum_{p \in \mathcal{C}} a_{s,p} x_p \leq \tau = \frac{v}{u} \cdot \frac{\ln \frac{1+\theta}{v\delta}}{\ln(1+\theta v/u)}$$

that is,

$$\sum_{p\in\mathcal{C}} a_{s,p}x_p/\tau \le 1.$$

□

Lemma 2 *At end of algorithm,*

$$\sum_{p\in\mathcal{C}} x_p/\tau \ge \frac{\ln(v|S|\delta)^{-1}}{\tau\theta\rho}\cdot opt$$

where opt is the objective function value of optimal solution for the maximum lifetime connected coverage problem and $\tau = (v/u)\log_{1+\theta v/u}\frac{1+\theta}{\delta v}$.

Proof Let us denote the initial value of x_p and y_s by $x_p(0)$ and $y_s(0)$. Denote by $x_p(i)$ and $y_s(i)$ the value of x_p and y_s after the ith iteration. Denote by $s^*(i)$ and $p^*(i)$ the value of s^* and p^* in the ith iteration. Furthermore, denote $X(i)=\sum_{p\in\mathcal{C}} x_p(i)$ and $Y(i)=\sum_{s\in S} y_s(i)$. Then, for $i\ge 1$, we have

$$\begin{aligned} Y(i) &= \sum_{s\in S} y_s(i-1) + \theta\frac{1}{a_{s^*(i),p^*(i)}}\sum_{s\in S} a_{s,p^*(i)}y_s(i-1)\\ &\le Y(i-1)+\theta(X(i)-X(i-1))\rho\min_{p\in\mathcal{C}}\sum_{s\in S} a_{s,p}y_s(k-1). \end{aligned}$$

It follows that

$$Y(i)\le Y(0)+\theta\rho\sum_{k=1}^{i}((X(k)-X(k-1))\min_{p\in\mathcal{C}}\sum_{s\in S} a_{s,p}y_s(k-1).$$

Note that opt is also the objective function value of optimal solution for the dual linear programming. Therefore,

$$opt = \min_{y_s}\frac{\sum_{s\in S} y_s}{\min_{p\in\mathcal{C}}\sum_{s\in S} a_{s,p}y_s},$$

where the minimiation is taken over $y_s\ge 0$ for $s\in S$. Thus,

$$\min_{p\in\mathcal{C}}\sum_{s\in S} a_{s,p}y_s(k-1)\le\frac{Y(k-1)}{opt}.$$

Hence

$$Y(i)\le |S|\delta+\frac{\theta\rho}{opt}\sum_{k=1}^{i}(X(k)-X(k-1))Y(k-1).$$

Define

$$w(0)=|S|\delta$$

and

$$w(i)=|S|\delta+\frac{\theta\rho}{opt}\sum_{k=1}^{i}(X(k)-X(k-1))w(k-1).$$

It is easy to prove by induction on i that $Y(i) \le w(i)$. Moreover,

$$\begin{aligned} w(i) &= \left(1 + \frac{\theta\rho}{opt}(X(i) - X(i-1))\right) w(i-1) \\ &\le e^{\frac{\theta\rho}{opt}(X(i)-X(i-1))} w(i-1) \\ &\le e^{\frac{\theta\rho}{opt}X(i)} w(0) \\ &= e^{\frac{\theta\rho}{opt}X(i)} |S|\delta. \end{aligned}$$

Suppose the algorithm stops at the mth iteration. Then $Y(m) \ge 1/v$. Hence

$$1/v \le Y(m) \le w(m) \le |S|\delta e^{\frac{\theta\rho}{opt}X(m)}.$$

Therefore,

$$\frac{opt}{X(m)/\tau} \le \frac{\tau\theta\rho}{\ln(v|S|\delta)^{-1}}.$$

□

Theorem 3 *If the problem* (1) *has a polynomial-time* ρ*-approximation, then the maximum lifetime connected coverage problem has a polynomial-time* $\rho(1+\varepsilon)$*-approximation for any* $\varepsilon > 0$.

Proof Choose $\delta = (1+\theta)((1+\theta)|S|)^{-\theta}/v$. Then

$$\frac{\ln\frac{1+\theta}{\delta v}}{\ln(\delta v|S|)^{-1}} = \frac{1}{1-\theta}.$$

Moreover, $(1+\theta v/u)^{u/(v\theta)+1} > e$, i.e., $\ln(1+\theta v/u) > \frac{v\theta}{u+v\theta}$. Thus,

$$\frac{\tau\theta\rho}{\ln(v|S|\delta)^{-1}} = \frac{(v/u)\theta\rho}{(1-\theta)\ln(1+\theta v/u)} \le \rho \cdot \frac{1+\theta(v/u)}{1-\theta}.$$

Choose θ such that

$$\frac{1+\theta v/u}{1-\theta} < 1+\varepsilon.$$

Then

$$\frac{opt}{\sum_{p\in C} x_p/\tau} \le (1+\varepsilon)\rho.$$

Next, we estimate the running time of the algorithm. Since ρ-approximation solution p^* for the problem (1) is assumed to be polynomial-time computable, every iteration can be carried out in polynomial-time. Therefore, it suffices to estimate the number of iterations. Note that at each iteration, at least one of y_s has its value increased. In the proof of Lemma 1, we showed that when the algorithm ends, each y_s has its value increased at most $\log_{1+\theta v/u}\frac{1+\theta}{\delta v}$ times. Therefore, the number of iterations is at most

$$|S|\log_{1+\theta v/u}\frac{1+\theta}{\delta v} = \frac{|S|\theta\ln((1+\theta)|S|)}{\ln(1+\theta v/u)} = O(|S|\log|S|)$$

for $\delta v = (1+\theta)((1+\theta)|S|)^{-\theta}$ and θ is fixed as ε is fixed. □

In next section, we will show that the problem (1) has polynomial-time $(7.875 + \varepsilon)$-approximations for any $\varepsilon > 0$ when all targets and all sensors lie in the Euclidean plane and all sensors have the same sensing radius R_s and the same communication radius R_c with $R_c \geq 2R_s$. The following is obtained by Theorem 3 and this fact.

Theorem 4 *The maximum lifetime connected coverage problem has polynomial-time $(7.875 + \varepsilon)$-approximations for any $\varepsilon > 0$ when all targets and all sensors lie in the Euclidean plane and all sensors have the same sensing radius R_s and the same communication radius R_c with $R_c \geq 2R_s$.*

3 Weighted connected coverage

Note that every $p \in \mathcal{C}$ is a connected coverage. The problem (1) is actually a weighted connected coverage problem in which each sensor s has two weights uy_s and vy_s for s in full-active phrase and semi-active phrase, respectively. In this section, we study approximations for the problem (1) by accumulating existing results in the literature. To do so, we first show the existence of polynomial-time $(4 + \epsilon)$-approximation for the *weighted coverage* problem as follows: Consider a set T of targets and a set S of sensors in the Euclidean plane. All sensors have the same sensing radius $R_s = 1$. Each sensor a nonnegative weight. Weights for different sensors may be different. The problem is to find a minimum-weight subset of sensors such that all targets are sensed by at least one chosen sensor.

There are two interesting special cases. In the first special case, all targets lie in a squre Q with edge length $\sqrt{2}/2$ and all sensors lie outside of the square Q. Huang et al. [9] proved an important property of optimal solution in this case.

Lemma 5 *Let A, B, L, R be four areas outside of Q as shown in Fig. 1. Then for each optimal solution Opt_{wc}, there exist four target locations a, b, c, d such that every target lying $\Delta_{up}(a, b) \cup \Delta_{low}(c, d)$ is covered by sensors in $Opt_{wc} \cap (A \cup B)$ and every target lying not in $\Delta_{up}(a, b) \cup \Delta_{low}(c, d)$ is covered by sensors in $Opt_{wc} \cap (L \cup R)$ where $\Delta_{up}(a, b)$ is the intersection of square Q and a facing-upper right angle with two edges parallel to diagonals and passing through points a and b, respectively and $\Delta_{low}(c, d)$ is the intersection of square Q and a facing-down right angle with two edges parallel to diagonals and passing through c and d, respectively.*

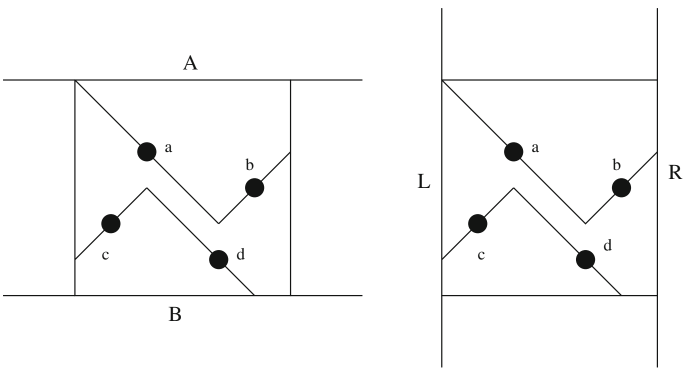

Fig. 1 Areas A, B, L, R and $\Delta_{up}(a, b)$ and $\Delta_{low}(c, d)$ in Lemma 5

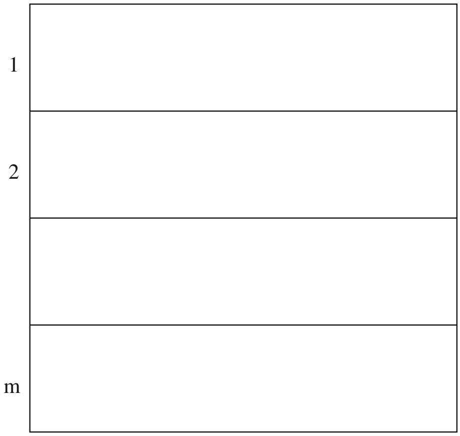

Fig. 2 m horizontal stripes in Lemma 5

In the second special case, all targets lie in a horizontal stripe and every sensor lies either above the stripe or below the stripe. In this case, Ambühl et al. [1] showed the existence of polynomial-time algorithm to compute the optimal solution. Motivated from this fact, Erlebach and Mihalák [6] showed the following result.

Lemma 6 *Consider m horizontal stripes each with width $\sqrt{2}/2$ as shown in Fig. 2 for a constant m. Suppose all targets lie in those stripes and all sensors have sensing radius $R_s = 1$. Then there exists a polynomial-time 2-approximation for the weighted coverage problem with constraint that each target is allowed to be covered only by sensors lying either above or below the stripe where the target lies.*

With above lemmas, we are ready to show the following.

Theorem 7 *There exists a polynomial-time $(4+\varepsilon)$-approximation for the weighted coverage problem in case that all targets and all sensors lie in the Euclidean plane and all sensors have the same sensing radius $R_c = 1$.*

Proof First, consider a fixed integer $m > 0$ and an $m \times m$ grid, called a *block*, consisting of m^2 cells each of which is a square with edge length $\sqrt{2}/2$. Suppose all targets lie in a block. In this case, we show that there exists a polynomial-time 4-approximation for the weighted coverage problem.

For simplicity, we will employ "guess" in our statement. We will make at most $4m^2 + 1$ guesses, each from a pool of polynomial size. Therefore, the number of possibilities for the guessed result is bounded by a polynomial and hence we may implement those guesses by enumerating all possibilities in polynomial-time.

Let Opt_{wc} be an optimal solution of the weighted coverage problem. We first guess the set $\mathcal{Q}$ of all cells each not containing any sensor in Opt_{wc}.

For each $Q \notin \mathcal{Q}$, we guess a sensor $s(Q) \in Opt \cap Q$. It is clear that $s(Q)$ covers all targets in Q. Denote $S' = \{s(Q) \mid Q \notin \mathcal{Q}\}$. Also, denote by T' the set of all targets not covered by any $s(Q) \in S'$.

For each $Q \in \mathcal{Q}$, we guess four points a, b, c, d and denote

$$H(Q) = T' \cap (\Delta_{up}(a, b) \cup \Delta_{low}(c, d)),$$
$$V(Q) = T' \cap (Q - (\Delta_{up}(a, b) \cup \Delta_{low}(c, d)).$$

Consider the $m \times m$ block as union of m horizontal stripes each consisting of m cells. Then $Opt_{wc} - S'$ covers all targets in $\cup_{Q \in \mathcal{Q}} H(Q)$ under constaint that each target is allowed to be covered by a sensor lying either above or below the stripe containing the target. By Lemma 6, within a polynomial-time, we can find a subset $S(H)$ of sensors such that

$$weight(S(H)) \leq 2 \cdot weight(Opt_{wc} - S')$$

and $S(H)$ covers all targets in $\cup_{Q \in \mathcal{Q}} H(Q)$. Similarly, in polynomial-time, we can find a subset $S(V)$ of sensors such that

$$weight(S(V)) \leq 2 \cdot weight(Opt_{wc} - S')$$

and $S(V)$ covers all targets in $\cup_{Q \in \mathcal{Q}} V(Q)$. Now, $S(H) \cup S(V) \cup S'$ covers all targets and their total weight is at most

$$4 \cdot weight(Opt_{wc} - S') + weight(S') \leq weight(Opt_{wc}).$$

Finally, by employing double partition (see [4]) and shifting techniques [2,8], we can obtain a polynomial-time $(4 + \varepsilon)$-approximation for the weighted coverage problem in the case described in theorem. □

A graph is called a *unit disk graph* if all nodes lie in the Euclidean plane and two nodes have an edge between them if and only if their distance is at most one. Given a unit disk graph $G = (V, E)$ with nonnegative node-weight $w : V \to R^+$s, and a node subset P, find the minimum-weight node subset U such that $P \cup U$ induced a connected subgraph, i.e., G contains a subtree with node set $P \cup U$. This is called the *node-weighted Steiner tree* problem in unit disk graphs. Zou et al. [16] showed the following.

Lemma 8 *The node-weighted Steiner tree problem in unit disk graphs has a polynomial-time 3.875-approximation.*

Zhang and Hou [14] first study the connected coverage in case that the communication radius is at least twice of sensing radius and showed that in such a case, for area coverage, the coverage implies the connectivity, i.e., if the required area is covered, then selected sensors induce a connected subgraph of input wireless sensor networks. This is not true for target coverage. However, with Lemma 8, we can still obtain a nice result on the problem (1) in this case.

Theorem 9 *The problem* (1) *has polynomial-time* $(7.875 + \varepsilon)$*-approximations for any* $\varepsilon > 0$ *when all targets and sensors lie in the Euclidean plane and all sensors have the same sensing radius* R_s *and the same communication radius* R_c *with* $R_c \geq 2R_s$.

Proof Let A be a $(4 + \varepsilon)$-approximation solution produced by a polynomial-time algorithm for the weighted coverage problem with weight $y_s u$ for each sensor s. Then

$$\sum_{s \in A} y_s u \leq (4 + \varepsilon) opt_{(1)}$$

where $opt_{(1)}$ is the objective function value of optimal solution for the problem (1). Since $R_c \geq 2R_s$, $A \cup Opt_{(1)}$ induces a connected subgraph of input wireless sensor network. Now, assign each sensor s with weight $y_s v$, find a polynomial-time 3.875-approximation solution B for the node-weighted Steiner tree problem. Then

$$\sum_{s \in B} y_s v \leq 3.875 \cdot \sum_{s \in Opt_{(1)}} y_s v \leq 3.875 \cdot opt_{(1)}$$

Therefore,

$$\sum_{s \in A} y_s u + \sum_{s \in B} y_s v \leq (7.875 + \varepsilon) opt_{(1)}.$$

□

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