

Efficient Algorithms for Topology Control Problem with Routing Cost Constraints in Wireless Networks

Ling Ding, *Student Member, IEEE*, Weili Wu, *Member, IEEE*, James Willson, *Student Member, IEEE*, Hongjie Du, *Student Member, IEEE*, Wonjun Lee, *Senior Member, IEEE*, and Ding-Zhu Du, *Member, IEEE*

Abstract—Topology control is one vital factor to a wireless network's efficiency. A *Connected Dominating Set* (CDS) can be a useful basis of a backbone topology construction. In this paper, a special CDS, named α *Minimum Routing Cost CDS* (α -MOC-CDS), will be studied to improve the performance of CDS based broadcasting and routing. In this paper, we prove that construction of a minimum α -MOC-CDS is NP-hard in a general graph and we propose a heuristic algorithm for construction of α -MOC-CDS.

Index Terms—Connected dominating set; routing path; wireless network; obstacle; general graph; NP-hard; topology control;

1 INTRODUCTION

The topology of a wireless network is an especially important factor to influence the performance of the network in terms of broadcasting, scheduling of transmission, and routing. On the other hand, some network links cannot benefit to packet routing and may even introduce redundancy and collisions in a network [1], [2]. Thus, many researchers focus on removing such kind of useless topology information from a network which is called topology control or topology management. It is believed that topology control can dramatically improve a network's broadband utilization and delivery ratio, extend network lifetime, and reduce interference [3] as well as packet retransmission [4].

According to [1], topology control strategies are categorized into two types — power control and hierarchical topology organization. The target of power control is to adjust nodes' transmission range to achieve balanced connectivity, while hierarchical topology organization aims to find a communication backbone from the original network in charge of all forwardings in the network. Routing information is only kept in the virtual backbone, so that routing path search time and communication cost will decrease greatly. *Connected Dominating Set* (CDS) is regarded as a workable and effective approach to hierarchical topology organization. In this paper, we will study how to construct a CDS to better a network's performance.

Since CDS can benefit much to wireless networks in many applications (such as broadcasting and routing),

study on CDS [5], [6], [7] has never stopped since it was touched in the first place. A network can be modeled as a bidirectional graph denoted as $G = (V, E)$ where V represents the set of nodes in the network while E represents the set of all links. A subset S of V is a CDS when S meets two constraints — 1). $\forall u \in V \setminus S, \exists v \in S$ having $(u, v) \in E$, 2). The induced graph $G[S]$ is connected.

When CDS is constructed, only nodes in CDS may forward data. In broadcasting [8], nodes in CDS can help spread data to the whole network. In routing, data will be sent to CDS and be delivered via nodes in CDS.

Thus, how to construct a CDS is closely related to the performance of CDS-based broadcasting and routing. If the size of CDS becomes larger, the number of nodes involved in forwarding will become larger correspondingly. As a result, redundancy and interference will increase. If the size of CDS is too small, some characteristics in original networks may be missing. For example, in CDS-based routing, the property of shortest path in the original network may not exist in the induced subnetwork by CDS. Hence, routing paths through CDS may be longer than that in the original network. In wireless networks, longer routing paths will cause a low delivery ratio. In Fig. 1(a), nodes (D, E, F) construct a minimum CDS. The shortest path between A and C in the original network is $p_{AC} = \{A, B, C\}$ of length 2. However, routing path between A and C through the minimum CDS will become $p'_{AC} = \{A, D, E, F, C\}$ of length 4 which is twice as that of p_{AC} . Obviously, it is more efficient to use p_{AC} for the routing between A and C .

To achieve efficient broadcasting and routing, CDS's size should be as small as possible while the routing paths' length does not increase a lot through the nodes in CDSs. Similar work has been done in [9] and [10]. The concept of *diameter* was defined to evaluate the length of

- L. Ding, W. Wu, J. Willson, H. Du and D.-Z. Du are with the Department of Computer Science, The University of Texas at Dallas, Richardson, TX, 75080. D.-Z. Du is also a WCU professor at Korea University. E-mail: {ling.ding, weiliwu, jkw053000, hxd061000, dzdu}@utdallas.edu
- W. Lee is with the Department of Computer Science & Engineering, Seoul, Korea University, Republic of Korea E-mail: wlee@korea.ac.kr

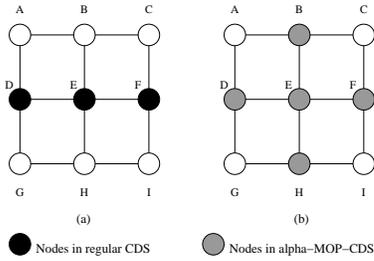


Fig. 1. Illustration of regular CDS and MOC-CDS. (a) A minimum Regular CDS. (b) A minimum 1-MOC-CDS.

the longest shortest path between any pair of nodes in a network in [9]. Since worst case cannot represent the performance throughout the network well, [10] proposed another concept named *Average Backbone Path Length* (ABPL). However, they did not touch the research on CDS with strict constraint of path length. In [11], they proposed MOC-CDS which is a CDS with shortest path constraint. However, the CDS's size quite large. They failed to figure out the tradeoff between CDS's size and routing cost.

To achieve efficient broadcasting and routing in wireless networks, a special CDS is defined and discussed in this paper, named as α Minimum rOuting Cost Connected Dominating Set (α -MOC-CDS). Besides all properties of regular CDS, α -MOC-CDS also has a special constraint — for any pair of nodes, there is at least one path all intermediate nodes on which belong to α -MOC-CDS and the number of the intermediate nodes is smaller than α times of that on the shortest path in the original network. Thus, routing hops will not increase too much. In Fig. 1 (a), all black nodes also construct a 3-MOC-CDS. Fig. 1 (b) shows a good example for the case $\alpha = 1$. Nodes $\{B, D, E, F, H\}$ construct a minimum 1-MOC-CDS. The shortest path between A and C is of length 2 in the network. The routing path between A and C through the 1-MOC-CDS is still of length 2 — $\{A, B, C\}$. From Fig. 1, we can find that there exists a tradeoff between CDS's size and routing hops.

In this paper, we also propose another concept of α -2hop-DS. And we prove that α -2hop-DS and α -MOC-CDS are equivalent to each other.

Our contributions include four aspects in this paper, as shown below:

- 1) To improve performance of a wireless network, a special CDS problem (α -MOC-CDS) with routing hop constraint is raised because CDS is a key technique of topology control.
- 2) We prove that the α -MOC-CDS is NP-hard.
- 3) We also prove that in polynomial time, it is impossible to find an algorithm to construct a 1-MOC-CDS with approximation ratio of $\rho \ln \delta$ in terms of CDS size, unless $NP \subseteq DTIME(n^{O(\log \log n)})$, where ρ is an arbitrary positive number ($\rho < 1$) and δ is the maximum degree in a graph.
- 4) We propose a heuristic strategy to select an α -MOC-CDS in polynomial time. And we prove that when $\alpha = 1$, the size of the selected 1-MOC-CDS

is within $(H(\delta(\delta - 1)/2) * OPT)$, where δ is the maximum degree in the graph, H is a harmonic function, and OPT represents the size of the minimum 1-MOC-CDS.

The rest of the paper is organized as follows: in Section 2, some related work on CDS will be reviewed. In Section 3, formal definition of α -MOC-CDS and α -2hop-DS will be given. We will prove they are equivalent to each other and that α -MOC-CDS is NP-hard. In Section 4, we introduce a heuristic local algorithm for selecting an α -MOC-CDS. And in Section 5, we prove the unreachable lower bound for 1-MOC-CDS and we also prove the approximation ratio of our algorithm. In Section 6, our algorithm is compared to other algorithms. The results show that our algorithm outperforms in terms of CDS's size and routing hops. Simulations also demonstrate that there is a tradeoff between CDS's size and the routing cost. Finally, the paper is concluded in Section 7.

2 RELATED WORK

Topology control or topology management has always been a hot topic in wireless networks since it has close relation to the performance of the control algorithms used in networks for scheduling of transmissions, routing, and broadcasting. In [1], Bao *et al.* divided topology control into two categories — one is power control and the other one is hierarchical topology organization. Power control adjusts the power on every node to ensure the connectivity of the network and balance the one-hop neighbor connectivity [12]. Hierarchical topology control aims to select a subset of nodes in the network serving as backbone over which essential network control functions are supported [13]. In hierarchical topology organization, CDS acts well as a backbone.

The research work on selecting a minimum CDS has never been interrupted because of its dramatic contributions to wireless networks. It is also well-known that computation of a minimum CDS in a general graph is an NP-hard problem [14] and it is even an NP-hard problem in *Unit Disk Graph* (UDG) [15]. Thus, much work has been devoted to achieving a better approximation ratio in polynomial time.

We can category CDS algorithms into two types — one is 2-stage and the other one is 1-stage. The 2-stage algorithms can also be divided into two categories. The main idea of the first category is to construct a *Dominating Set* (DS) and add more nodes to make the selected DS connected. As a result, a CDS is constructed. In [16], a 2-stage strategy is proposed yielding an approximation ratio of $H(\delta) + 2$, where H is a harmonic function. Based on the ideas of Independent Set in [17] and Steiner Tree in [18], Min *et al.* [19] proposed an algorithm constructing a CDS with size smaller than $6.8|OPT|$, where OPT is the size of the minimum CDS. The other category is to construct a redundant CDS firstly, then remove some nodes to get a smaller CDS. A typical algorithm of this type is that proposed in [20].

The main idea of 1-stage algorithms is to construct a CDS directly skipping any intermediate step. In [16], an 1-stage strategy is proposed with approximation ratio of $2H(\delta) + 2$. Based on the main idea of the 2-stage algorithm in [16], Ruan *et al.* [21] made a modification of the selection standard of DS. Therefore, 2-stage is reduced to 1-stage, with approximation ratio of $3 + \ln(\delta)$.

In [22], [23], the existence of obstacles were considered and they assumed that every node in the same network shared the same maximum transmission range. Hence, they modeled the network as a Quasi Unit Disk Graph (QUDG). However, due to the fact that not all nodes are homogenous in a network [24], different nodes may have different transmission ranges. Therefore, [24] studied CDS in Disk Graph (DG). Hence, it is more reasonable to model the wireless network as a general graph, instead of QUDG, UDG, or DG.

3 PROBLEM STATEMENT

Different from QUDG, we assume that nodes in the network may have different transmission ranges. Similarly, we also consider the existence of obstacles. Therefore, we model the network as a general graph $G = (V, E)$, where V represents the node set in the network and E represents an edge set including all direct links in the network. In this paper, we denote the path between any pair of nodes u and v as $p(u, v) = \{u, w_1, w_2, \dots, w_k, v\}$ where w_i ($1 \leq i \leq k$) represents the intermediate node on $p(u, v)$. The shortest path between u and v is the path with the smallest number of intermediate nodes, represented by $p_{shortest}(u, v)$. And the distance between u and v is equal to the hop count on the shortest path between u and v , denoted as $Dist(u, v) = |p_{shortest}(u, v)| - 1$. We define our α -MOC-CDS as follows:

Definition 1 (α -MOC-CDS). *The α Minimum rOuting Cost Connected Dominating Set (α -MOC-CDS) is a node set $D \subseteq V$ such that*

- 1) $\forall u \in V \setminus D, \exists v \in D$, such that $(u, v) \in E$.
- 2) *The induced graph $G[D]$ is connected.*
- 3) $\forall u, v \in V$, if $Dist(u, v) > 1$, then $\exists p^D(u, v)$ on which all intermediate nodes belong to D and $m_D(u, v) \leq \alpha * m(u, v)$, where $m_D(u, v)$ and $m(u, v)$ are the number of intermediate nodes on $p^D(u, v)$ and $p_{shortest}(u, v)$ respectively.

If the distance between the source and the destination is 1, then the messages can be delivered directly. To simplify the problem of α -MOC-CDS, we find an equivalent problem α -2hop-DS.

Definition 2 (α -2hop-DS). *The α -2hop Minimum rOuting Cost Dominating Set (α -2hop-DS) is a node set $D \subseteq V$ such that*

- 1) $\forall u \in V \setminus D, \exists v \in D$, such that $(u, v) \in E$.
- 2) $\forall u, v \in V$, if $Dist(u, v) = 2$, then $\exists p^D(u, v)$ on which all intermediate nodes belong to D and $m_D(u, v) \leq \alpha * m(u, v)$, where $m_D(u, v)$ and $m(u, v) = 1$ are

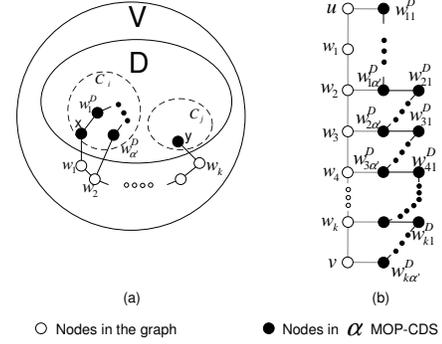


Fig. 2. Equivalence between α -MOC-CDS and α -2hop-DS.

the number of intermediate nodes on $p^D(u, v)$ and $p_{shortest}(u, v)$ respectively.

Next, we will prove that α -MOC-CDS and α -2hop-DS are equivalent to each other in Theorem 1.

Theorem 1. *A node subset $D \subseteq V$ of a general graph $G = (V, E)$ is an α -MOC-CDS if and only if D is an α -2hop-DS.*

Proof: (1). If a node subset D is an α -MOC-CDS, definitely it is also an α -2hop-DS because D must meet the two constraints of Def. 2.

(2). Suppose a node subset D is an α -2hop-DS, then we will prove that the node subset D must meet the three constraints of α -MOC-CDS (Def. 1) step by step. (a). "Dominating". Because of the first constraint of Def. 2, we can get that an α -2hop-DS D must also meet the first constraint in Def. 1. (b). "Connectivity". Assume that the induced subgraph $G[D]$ by D is disconnected, then there must be at least two connected components in D . Denote every connected components in $G[D]$ as C_i in Fig. 2 (a). Find one pair of nodes $x \in C_i$ and $y \in C_j$ where $i \neq j$, such that the distance between x and y is the shortest among those pairs of nodes from different connected components. Choose one path $p(x, y) = \{x, w_1, w_2, \dots, w_k, y\}$ and we have $w_m \in V \setminus D$, where $1 \leq m \leq k$. Thus, for the pair of nodes x and w_2 of distance 2, there is no such a path $p = \{x, w_1^D, \dots, w_{\alpha'}^D, w_2\}$, where $w_m^D \in D$, $1 \leq m \leq \alpha'$ and $\alpha' < \alpha$ (Or x and y cannot be the pair from different components with the shortest distance because if p exists, then $Dist(w_{\alpha'}^D, y) \leq Dist(x, y)$). As a result, there is a case violates Def. 2. Thus, we can get that D is not an α -2hop-DS. Hence, assumption violates the fact that D is an α -2hop-DS. Therefore, "Connectivity" is proven. (c). " α -Constraint". Given any pair of nodes u and v , there must be a shortest path $p_{shortest}(u, v) = \{u, w_1, w_2, \dots, w_k, v\}$ in G . Because that D is an α -2hop-DS, we can find some nodes in D to replace w_1 connecting u and w_2 , and the number of nodes found in D is at most α in Fig. 2 (b). Thus, we can get a new path $p'(u, v) = \{u, w_{1\alpha'}^D, \dots, w_{1\alpha_1}^D, w_2, \dots, w_k, v\}$, where $w_{1m}^D \in D$, $\forall 1 \leq m \leq \alpha'_1$ and $\alpha'_1 \leq \alpha$. Similarly, we can find nodes from D to replace w_2 to form a new path $p''(u, v) = \{u, w_{1\alpha_1}^D, \dots, w_{1\alpha_1}^D, w_{2\alpha_2}^D, \dots, w_{2\alpha_2}^D, w_3, \dots, w_k, v\}$. Finally, we can get a path $p^D(u, v) =$

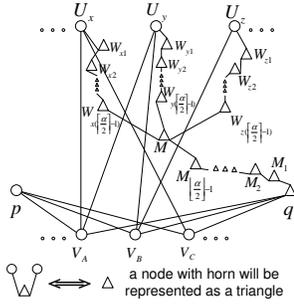


Fig. 3. Reduction from DECISION VERSION OF SET COVER to DECISION VERSION OF α -2HOP-DS.

$\{u, w_{11}^D, \dots, w_{1\alpha_1}^D, w_{21}^D, \dots, w_{2\alpha_2}^D, w_{k1}^D, \dots, w_{k\alpha_k}^D, v\}$ where all intermediate nodes belong to D . During the process, we find at most α nodes to replace each w_m where $1 \leq m \leq k$. Thus $m_D(u, v) \leq \alpha * m(u, v)$. Therefore, “ α -Constraint” is proven.

In sum, equivalence is proven. \square

Theorem 2. The α -2hop-DS is NP-hard, $\forall \alpha \geq 1$.

Proof: It suffices to show the following decision version of α -2HOP-DS is NP-hard.

DECISION VERSION OF α -2HOP-DS: Given a graph G and a positive integer k , determine whether G has an α -2hop-DS of size at most k .

To do so, we reduce the following decision version of set cover problem to DECISION VERSION OF α -2HOP-DS.

DECISION VERSION OF SET COVER: Given a collection \mathcal{C} of subsets of a finite set X and a positive number h , determine whether \mathcal{C} contains a set cover \mathcal{C}' of at most h subsets.

In Fig. 3, for each element $x \in X$, create a node U_x and for each subset $A \in \mathcal{C}$, create a node V_A . Connect U_x and V_A with an edge (U_x, V_A) if and only if $x \in A$. We now obtain a bipartite graph H . For further construction, let us first define what is a horn-node.

An edge in a graph is called a *free* edge if it has an endpoint with degree one. If a node u is incident to two free edges, we say that u is a *horn-node*. Note that in any graph, a horn-node belongs to any α -2hop-DS.

Next, we construct a graph G in following way based on bipartite graph H .

(a) Set a new node p (without horn). For every $A \in \mathcal{A}$, connect p to node V_A with an edge (p, V_A) .

(b) Set a new horn-node q . $\forall A \in \mathcal{C}$, connect q to node V_A with edge (q, V_A) . For the case $\alpha = 1$, connect q to U_x while for other case, set another horn-node M . Connect q to M by a path having $\lfloor \alpha/2 \rfloor - 1$ horn-nodes $M_1, M_2, \dots, M_{\lfloor \alpha/2 \rfloor - 1}$ (when $\alpha = 2$ or $\alpha = 3$, connect q and M directly). For every $x \in X$, connect M to node U_x with a path having $\lfloor \alpha/2 \rfloor - 1$ horn-nodes — $W_{x1}, \dots, W_{\lfloor \alpha/2 \rfloor - 1}$ (when $\alpha = 2$, connect M to U_x directly).

Let r be the number of horn-nodes in above construction. Then r is a polynomial function of $|X|$. Set $k = h + r$. We will show that G has an α -2hop-DS of size at most k if and only if \mathcal{C} contains a set cover of at most h subsets.

First, assume \mathcal{C} contains a set cover \mathcal{C}' of at most h subsets. Let $D = \{V_A \mid A \in \mathcal{C}'\} \cup \{\text{all nodes with horns}\}$.

Then $|D| = k$ and it is easy to check that D is an α -2hop-DS. Indeed, there are four types of pairs of nodes with distance two. Each of them is connected with a path with at most α intermediate nodes in D as follows.

(1) For any pair of nodes in the neighbor of a horn-node, they are connected by the horn-node.

(2) For any pair of nodes in the neighbor of a node U_x for some $x \in X$, we have constructed a path with at most α intermediate horn-nodes, connecting them. For example, for the pair (V_A, W_{x1}) , they are connected by the path $\{W_{x1}, \dots, W_{x(\lfloor \alpha/2 \rfloor - 1)}, M, M_{\lfloor \alpha/2 \rfloor - 1}, \dots, M_2, M_1, q, V_A\}$ of $\alpha - 1$ intermediate nodes.

(3) For any pair of nodes in the neighbor of p , they are also in the neighbor of q and hence are connected by q .

(4) For any pair of nodes in the neighborhood of a node V_A for some $A \in \mathcal{C}$, we have three cases.

(4a) They are U_x and U_y for some $x, y \in X$. In this case, U_x and U_y are connected by a path with α horn-nodes. For example, in Fig. 3, U_x and U_y can be connected by the path $\{U_x, W_{x1}, \dots, W_{x(\lfloor \alpha/2 \rfloor - 1)}, M, W_{y(\lfloor \alpha/2 \rfloor - 1)}, \dots, W_{y1}, U_y\}$ with at most α intermediate nodes.

(4b) They are q and U_x for some $x \in X$. In this case, q and U_x are connected by a path with $\alpha - 1$ horn-nodes.

(4c) They are p and U_x for some $x \in X$. In this case, since \mathcal{C}' is a set cover, there exists $A \in \mathcal{C}'$ such that $x \in A$, so that p and U_x are connected by $V_A \in D$.

Next, assume that G has an α -2hop-DS D of size at most k . Since D must contain all horn-nodes, the number of nodes in D without horn is at most h . Note that for any pair of nodes in case (1)(2)(3)(4a)(4b), they are already connected by a path with at most α intermediate horn-nodes. Therefore, we need only to consider case (4c), i.e., node pairs p and U_x for $x \in X$. Note that if a path connecting p and U_x has at most α intermediate nodes, then it must contain an edge (V_A, U_x) for some $A \in \mathcal{C}$ with $x \in A$. Therefore, D must contain a node U_A with $x \in A$. It follows that $\{A \mid U_A \in D\}$ form a set cover and $|\{A \mid U_A \in D\}| \leq h$. \square

4 ALGORITHM DESCRIPTION

In this paper, we propose a heuristic localized algorithm to construct an α -2hop-DS. Every node v is selected based on its own $\alpha + 1$ local topology information, denoted as $G(v) = (V_v, E_v)$. $N(v)$ is used to record the neighbors of v in $G(v)$. From $G(v)$, we can derive a weighted subgraph $G'(v) = (V'_v, E'_v, W'_v)$, where $V'_v = V_v$ and $E'_v = E_v$. Initially, $\forall u, v \in V'_v$ having $(u, v) \in E'_v$, then the weight of the edge (u, v) is equal to 0 denoted as $w(u, v) = 0 \in G'(v)$, otherwise $w(u, v) = \infty$.

Based on the local topology, every node v will store a set *pair*(v) which is used to record the pairs of nodes $u \in V'_v$ and $w \in V'_v$ having $Dist(u, w) = 2 \in G(v)$, $p'(u, w) = \{u, v, w\} \in G'(v)$ and $w(u, v) + w(v, w) + 1 \leq \alpha$.

After calculating the set *pair* based on the topology in $G'(v)$, v will send out its *id* and *pair* set to all its neighbors in $N(v)$. Node v will list all its white neighbors

in $G(v)$ in a linear ordering of their priorities. For every node v , the priority $prio(v)$ is determined by two factors — $f(v) = |pair(v)|$ and $id(v)$. For any two nodes u and w , $prio(u) > prio(w)$ when and only when $[f(u) > f(w)]$ or $[f(u) = f(w), \text{ but } id(u) > id(w)]$.

Sending out Flags Condition: Node v will send a flag to node x if and only if $pair(x) \cap pair(y) = \emptyset, \forall x, y \in N(v)$ having $prio(y) > prio(x)$.

In our algorithm, every node can send more than one flag out at one time.

Edge Set Update: If Node v turns black, then for any pair of nodes u and w having $Dist(u, w) = 2 \in G'(v)$ and $p(u, w) = \{u, v, w\} \in G'(v)$, then edge (u, w) will be added to $G'(v)$ and $w(u, w) = w(u, v) + w(v, w) + 1$.

When one node v receives flags from all its neighbors in $N(v)$, then v will turn into black. v will update the weighted edges according to the Edge Set Update and propagate its $pair(v)$ and the new weighted edges $\alpha + 1$ hops away. Node w will update its $G'(w)$ according to the update information it receives. Add those pairs (x, y) to $pair(w)$ having $Dist(x, y) = 2 \in G'(w)$ and $w(x, w) + w(w, y) + 1 \leq \alpha \in G'(w)$. There are two kinds of pairs one node w needs to remove from $pair(w)$. The first one is to remove those pairs in $pair(v)$. The other one is to remove those pairs (x, y) having $Dist(x, y) = 2 \in G'(w)$ and $\exists z \in D, w(x, z) + x(z, y) + 1 \leq \alpha \in G'(w)$. Then the updated “ f ” values will be propagated again. The algorithm will stop when all nodes’ “ f ” values are 0.

Algorithm 1 Distributed Selection of α -2hop-DS

- Step 1.** Each node v with nonempty $pair(v)$, send $prio(v)$ and $pair(v)$ to its neighbors;
- Step 2.** Each node v will list all its white neighbors based on $G(v)$, in a linear ordering of their priorities. And send out flags on the policy of *Sending out Flags Condition*.
- Step 3.** If a node v receives flags from all its neighbors in $G(v)$, then change color to black, update the edges based on the policy Edge Set Update. Then send $pair(v)$ and edge update information to all of its neighbors within $\alpha + 1$ hops in $G'(v)$. Lastly, set $pair(v) = \phi$;
- Step 4.** If a white node w receives $pair(v)$ and updated edges, update its $G'(w)$ firstly. Add those pair x and y of distance 2 in $G(w)$ to $pair(w)$, where $w(x, w) + w(w, y) + 1 \leq \alpha \in G'(w)$. Then compute union of such $pair(v)$'s with U_w ($U_w = \phi$ from the beginning). U_w will also include those pair x and y of distance 2 in $G(w)$, where there exists a node z which has already been selected having $w(x, z) + w(z, y) + 1 \leq \alpha$. Update $pair(w)$ by setting $pair(w) \leftarrow pair(w) - U_w$.
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All black nodes selected in Alg. 1 will construct an α -2hop-DS. If the graph is a complete graph, then choose one node randomly as an α -2hop-DS.

In Fig. 1 (a), suppose $id(E) > id(D) > id(F) > id(B) > id(H) > id(A) > id(C) > id(G) > id(I)$

and $\alpha = 3$. In E 's view, $prio(D) > prio(F) > prio(B) > prio(H)$. E also knows $pair(B) \cap pair(D) \neq \phi$, where $pair(B) = \{(A, C), (A, E), (C, E)\}$ and $pair(D) = \{(A, G), (A, E), (E, G)\}$. As a result, E will not send a flag to B while it will send a flag to D . On the other hand, E will send a flag to F because $pair(F) \cap pair(D) = \phi$ even though $prio(D) > prio(F)$. Finally, at the first round D, E, F will collect flags from all their neighbors respectively. All of the three nodes will be selected into the CDS. Because D and F are selected, $w(G, E) = 1 \in W'(D)$ and $w(E, I) = 1 \in W'(F)$ should be added. D and F will propagate away their $pair(D), pair(F)$, and their new edges. When H collects the update from D and F , it will remove pairs (E, G) and (E, I) from $pair(H)$ because $(E, G) \in pair(D)$ and $(E, I) \in pair(F)$. The pair (G, I) will be removed from $pair(H)$, because we have $w(G, E) + w(E, I) + 1 = 3$. Therefore, we can get all nodes’ f value will be 0 after the first round. Therefore, if $\alpha = 3$, the CDS selected by our algorithm for Fig. 1 is $\{D, E, F\}$.

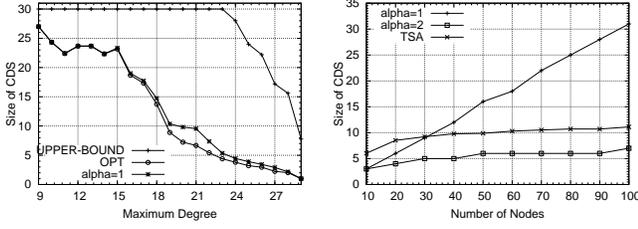
5 THEORETICAL ANALYSIS

In this section, we will prove that for the case $\alpha = 1$, there is an unreachable bound for 1-2hop-DS and the approximation ratio of Alg. 1 is $H(\delta * (\delta - 1)/2)$, where H is a harmonic function and δ is the maximum degree in the graph.

Theorem 3. 1-2hop-DS does not have a polynomial-time approximation algorithm with performance ratio $\rho \ln \delta, \forall \rho < 1$, where δ is the maximum node degree of input graph, unless $NP \subseteq DTIME(n^{O(\log \log n)})$, where $n = |X|$.

Proof: It has been proven in [25] that SET COVER does not have an polynomial-time approximation with performance ratio $\rho \ln n, \forall 0 < \rho < 1$ unless $NP \subseteq DTIME(n^{O(\log \log n)})$. It is important to note that this lower bound can be applied to the special case $|C| \leq n$. We show our theorem base on this fact.

Assume for contradiction that 1-2hop-DS has a polynomial-time algorithm with performance ratio $\rho \ln \delta$, for some $\rho < 1$. Given an instance of SET COVER, with $|C| \leq |X| = n$, construct an instance of 1-2hop-DS as in Theorem 2. By the same reasoning as in Theorem 2, we know the relation of the optimal solutions is $opt_{SC} + 1 = opt_{1-2hop-DS}$. In the constructed 1-2hop-DS instance, the node with greatest degree is q , with degree $|C| + |X| + 2$. Since $|C| \leq |X| = n$, we can find an approximate solution in polynomial time with size at most $\rho \ln(2n+2)opt_{1-2hop-DS}$, and hence an approximate solution to the original SET COVER instance with the same size. In terms of opt_{SC} , this is $\rho \ln(2n+2)(opt_{SC} + 1)$. For sufficiently large n and opt_{SC} , we have $\rho \ln(2n+2)(opt_{SC} + 1) \leq \frac{1}{2}(\rho + 1)(\ln n)opt_{SC}$. $\rho' = \frac{1}{2}(\rho + 1) < 1$, so this gives us an approximation algorithm for SET COVER with performance bound $\rho' \ln n$ for some $\rho' < 1$. This is impossible unless $NP \subseteq DTIME(n^{O(\log \log n)})$. Therefore either our original assumption is incorrect and there is no such algorithm, or $NP \subseteq DTIME(n^{O(\log \log n)})$. \square



(a) 30 nodes in a general graph (b) CDS' size comparison in DG
 Fig. 4. Bound of size of 1-MOC-CDS in General Graphs and comparison of CDS in DG.

Theorem 4. *If $\alpha = 1$, then Alg. 1 produces an approximation solution with performance ratio $H(\delta * (\delta - 1)/2)$.*

Proof: During the computation of the new distributed algorithm, if a node u is selected to join 1-MOC-CDS, we assign a weight $1/|pair(u)|$ to each pair (v, w) in $pair(u)$.

Suppose $\{u_1^*, u_2^*, \dots, u_k^*\}$ is an optimal solution D^* . We estimate total weight collected at each node u_i^* .

Initially, u_i^* has "f" value $f_0(u_i^*) = |pair(u_i^*)|$. After some nodes join the 1-MOC-CDS, $pair(u_i^*)$ is updated. Suppose for updated $pair(u_i^*)$, $f_1(u_i^*) = |pair(u_i^*)|$. $f_0(u_i^*) - f_1(u_i^*)$ is the number of pairs originally in $pair(u_i^*)$ and now are connected by those nodes currently selected in the 1-MOC-CDS. Each such pair (v, w) has distance 2 such that v and w are adjacent to u_i^* and also adjacent to a new node x in CDS. By condition that x joins the 1-MOC-CDS, at least one of v and w sends flag to x . This means that before update, $f_0(u_i^*) = |pair(u_i^*)| \leq |pair(x)|$. Therefore, (v, w) received weight $1/f_0(u)$, where $1/f_0(u) \leq 1/f_0(u_i^*)$. All $f_0(u_i^*) - f_1(u_i^*)$ pairs receives weight at most $(f_0(u_i^*) - f_1(u_i^*))/f_0(u_i^*)$.

Similarly, we can prove that during the computation of this distributed algorithm, all pairs in $pair(u_i^*)$ received total weight at most

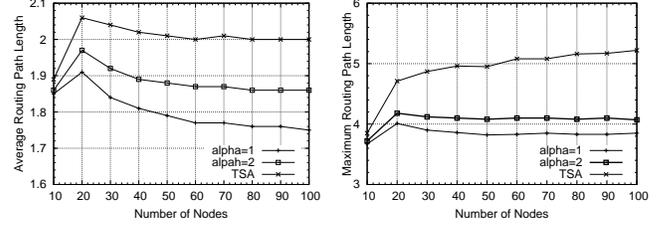
$$\sum_{i=0}^{i=k-1} (f_i(u_i^*) - f_{i+1}(u_i^*))/f_i(u_i^*) \leq H(\delta(\delta - 1)/2) \quad (1)$$

where H is the harmonic function, $f_k(u_i^*) = 0$ and δ is the maximum node degree of input graph.

Note that when one node is selected to D , the charged weight is 1. Thus, the total weight equals to the number of selected nodes, where D is the node set selected by Alg. 1. Therefore, we have $|D| \leq H(\delta(\delta - 1)/2)opt$, where opt is the size of the minimum 1-MOC-CDS. \square

6 SIMULATION

This part includes two subparts. The first one evaluates whether the size of 1-MOC-CDS obtained from Alg. 1 is under the upper bound we have already proved. The second one evaluates Alg. 1 by comparing α -MOC-CDS with traditional CDS without routing length constraint. We compare them in terms of *Maximum Routing Path Length* (MRPL) and *Average Routing Path Length* (ARPL). MRPL is defined as the maximum routing path length in the network, while ARPL is defined as the average



(a) Average Routing Path Length (b) Maximum Routing Path Length

Fig. 5. Comparison of Average Routing Path Length and Maximum Routing Path Length in DG Networks between Alg. 1 and TSA in Disk Graph.

length of routing paths in the network. In the simulations, except the source and destination, all nodes on the routing path should belong to CDS.

6.1 Simulation Environment

To evaluate Alg. 1, we check three types of networks. The first type is a network in which nodes may have different transmission ranges and obstacles may obstruct communications among nodes. This type is named General Network because this type can be modeled as a general graph. In General Networks, we show that 1-MOC-CDS derived from Alg. 1 is indeed under the upper bound we have proved before. The second type is a network in which different transmission ranges are allowed, however, obstacles are not considered. The second type is named DG Network since this type can be modeled as a disk graph. In DG Network, we will compare Alg. 1 with TSA [24]. The last one is an ideal one in which all nodes should have the same transmission ranges and no obstacle exists. This one is named UDG Network because it can be modeled as a unit disk graph. In UDG Network, Alg. 1 will be compared with CDS-BD-D [10], FKMS06 [26], and ZJH06 [27].

6.1.1 General Network

To simulate network of this category, n nodes are randomly deployed to a fixed area of $100m \times 100m$. To find the minimum size of 1-MOC-CDS in a given graph, we check whether there exists such a 1-MOC-CDS of size k where the initial value of k is 1 and it will increase one by one. If there exists such a subset meeting all requirements of 1-MOC-CDS, then we stop. Otherwise, increase k by 1 and check again. The corresponding k is the size of a minimum 1-MOC-CDS of the given graph when we stop. Since we have to use brute-force search, we can only get optimal solution when network size is limited (here, $n = 30$). For a certain n , the maximum degree of a network can vary from 1 to $n - 1$. Here, once we fix a certain n and a maximum degree, we generate 100 instances. Nodes are assigned a transmission range randomly. Definitely, we have to generate a connected network as our input. We take the average value among 100 instances as our results.

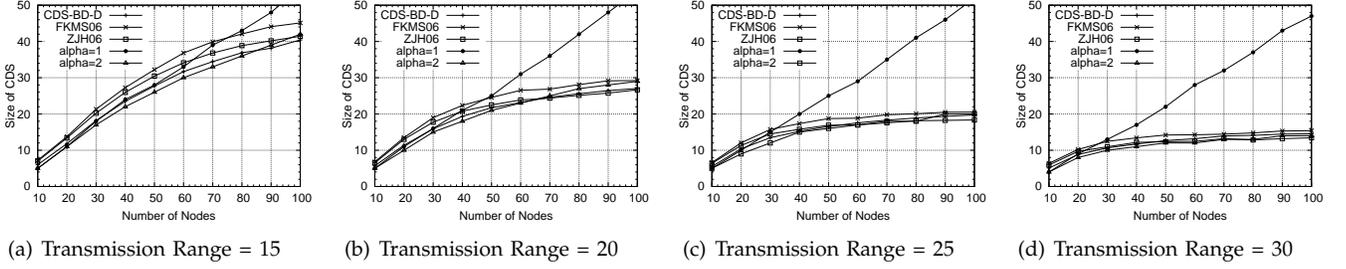


Fig. 6. Comparison of CDS's Size among CDS-BD-D, FKMS06, ZJH06, and Alg. 1 in UDG Networks.

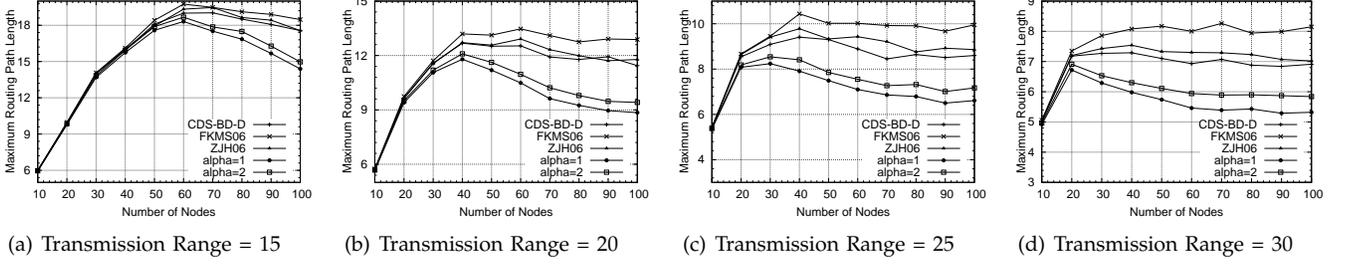


Fig. 7. Comparison of Maximum Routing Path Length among CDS-BD-D, FKMS06, ZJH06, and Alg. 1 in UDG Networks.

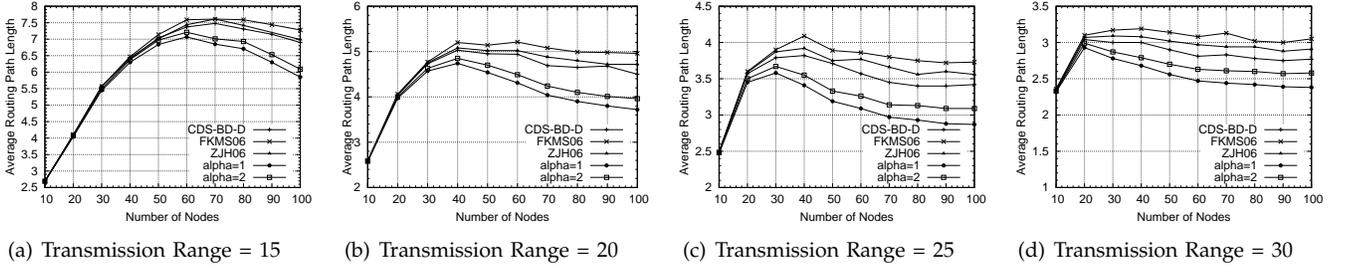


Fig. 8. Comparison of Average Routing Path Length among CDS-BD-D, FKMS06, ZJH06, and Alg. 1 in UDG Networks.

6.1.2 DG Network

To simulate networks of this category, n nodes are randomly deployed to a fixed area of $800m \times 800m$. n varies from 10 to 120 with increment of 10. Each node v is randomly assigned a transmission range $r \in [r_{min}, r_{max}]$, where $r_{min} = 200m$ and $r_{max} = 600m$. For each value of n , 1000 network instances are investigated. Results of the same n are averaged among 1000 instances.

6.1.3 UDG Network

To simulate network of this category, n nodes are deployed randomly in a fixed area of $100m \times 100m$ and all nodes have the same transmission range. n is incremented from 10 to 100 by 10, while transmission range varies among $15m$, $20m$, $25m$, and $30m$. For a certain n and transmission range, 100 instances are generated. Results are averaged among 100 instances.

6.2 Simulation Results

Fig. 4(a) shows that the size of 1-MOC-CDS selected by Alg. 1 strategy is significantly less than the upper bound and very close to that of the optimal solution. Note the bigger the maximum degree is, the smaller size of 1-MOC-CDS is. The reason is that a node with bigger

degree can be an intermediate node of more shortest paths between pairs of nodes, which can reduce the size of CDS greatly.

From Fig. 4(b), 5, 6, 7, and 8, we can get that there exists a tradeoff between α and routing cost — the size of 2-MOC-CDS is much smaller than that of 1-MOC-CDS, however, MRPL or ARPL of 1-MOC-CDS is much smaller than that of 2-MOC-CDS. The reason is that an α -MOC-CDS ($\forall \alpha \leq 1$), must be an $(\alpha + 1)$ -MOC-CDS, however, an $(\alpha + 1)$ -MOC-CDS may not be an α -MOC-CDS. In Fig. 1, the minimum size of 3-MOC-CDS is 3 while the minimum size of 1-MOC-CDS is 5.

Fig. 5 shows that the MRPL and the ARPL of Alg. 1 are smaller than those of TSA. This illustrates that our Alg. 1 can also work well in DG Network. In Fig. 5, when $\alpha = 1$, the ARPL of Alg. 1 is about 12.5% less than that of TSA while the MRPL of Alg. 1 is about 20% less than that of TSA, and the cost is the increase of size of CDS in Fig. 4(b). When $\alpha = 2$, the ARPL of Alg. 1 is still about 10% less than that of TSA while the MRPL of Alg. 1 is about 17% less than that of TSA. TSA tends to include nodes with larger transmission range in CDS. However, large transmission range does not necessarily imply big node degree which is a selection criteria of Alg. 1.

Fig. 7 and Fig. 8 show that Alg. 1 is also efficient in UDG Networks. As shown in Fig. 7 and Fig. 8, the MRPL of FlagContest is about 15%-40% better and the ARPL of Alg. 1 is around 10%-30% better, when the number of nodes exceeds 30 and no matter whether $\alpha = 1$ or $\alpha = 2$. Note ARPL and MRPL increase firstly and then decrease. The reason is that in a connected network with small size of nodes, the routing path length is more likely to increase when a new node is added. For example, a network with 1 node inside has ARPL equal to 0. When a new node is connected to the network, both ARPL and MRPL will increase to 1. Hence, routing path length increases when n increases (n is relatively small). However, when n exceeds a certain value, newly added nodes are more likely to make distance between nodes smaller and the network more connected (considering physical space is fixed). That's why there is a decrease when network size becomes big enough. In addition, when transmission range increases, networks are more connected considering physical space is fixed. It is trivial to conclude that routing path will decrease when transmission range increases, which explains both MRPL and ARPL decrease while transmission range increases as shown in Fig. 7 and Fig. 8.

7 CONCLUSION

In this paper, we study one technique (CDS) in hierarchical topology organization of wireless networks. To achieve efficient routing, a minimum routing cost preserving CDS named α -MOC-CDS is studied. Hence, we can achieve efficient routing even through CDS. We prove that the α -MOC-CDS is NP-hard in a general graph. An unreachable lower bound for construction of an α -MOC-CDS in polynomial time is proved in this paper as well. We propose a heuristic local algorithm to construct α -MOC-CDS in polynomial time. When $\alpha = 1$, the performance ratio of our algorithm is $H(\delta(\delta - 1)/2)$, where H is a harmonic function and δ is the maximum node degree in the network. The simulation results demonstrate that our algorithm outperform other compared algorithm. At the same time, from simulation, we can find there does exist a tradeoff between CDS's size and routing cost.

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Ling Ding received the BS degree in computer science from Nanjing University of Science and Technology, Nanjing, in 2004 and the MS degree in computer science from Shanghai Jiao Tong University, Shanghai, in 2008. She is currently a PhD student in Department of Computer Science, University of Texas at Dallas, Richardson. Her research interests include wireless networks and approximation algorithm design. She is a student member of IEEE.



Dr. Weili Wu received her MS and PhD degrees in computer science both from University of Minnesota, in 1998 and 2002 respectively. She is currently an associate professor and a lab director of the Data Communication and Data Management Laboratory at the Department of Computer Science and Engineering, the University of Texas at Dallas. Her research interest is mainly in database systems, especially with applications in Social networks, geographic information systems and bioinformatics,

distributed data-base in internet system, and wireless database systems with connection to wireless communication. She has published more than 90 research papers in various prestigious journals and conferences such as IEEE Transaction on Knowledge and Data Engineering (TKDE), IEEE Transaction on Multimedia, IEEE Transactions on Parallel and Distributed Systems (TPDS), ACM Transaction on Sensor Networks (TOSN), Theoretical Computer Science, Journal of Complexity, Discrete Mathematics, Discrete Applied Mathematics, IEEE ICDCS the International Conference on Distributed Computing Systems, IEEE INFOCOM The Conference on Computer Communications, ACM SIGKDD International Conference on Knowledge Discovery & Data Mining, SIAM Conference on Data Mining, International Conference on Database and Expert Systems Applications (DEXA), International Conference on Computer Science and Informatics, etc. She is an associate editor of International Journal of Bioinformatics Research and Applications (IJBRA), and International Journal of Knowledge and Information Systems (KAIS).



James Willson received his B.S. degree in 1999 from Washington University, and his M.S. at The University of Texas at Dallas in 2007. He is currently working towards his Ph.D. at The University of Texas at Dallas. His research interests include design and analysis of algorithms for combinatorial optimization problems in communication networks. He is a student member of IEEE.



Hongjie Du received his BS degree in computer science from Harbin Institute of Technology in 2004. He will receive the Ph.D degree in computer science from University of Texas at Dallas in 2010. His research interests include wireless networks, social networks and optimization theory. He is a student member of IEEE and ACM.



Wonjun Lee (M'00-SM'06) received B.S. and M.S. degrees in computer engineering from Seoul National University, Seoul, Korea in 1989 and 1991, respectively. He also received an M.S. in computer science from the University of Maryland, College Park, USA in 1996 and a Ph.D. in computer science and engineering from the University of Minnesota, Minneapolis, USA, in 1999. In 2002, he joined the faculty of Korea University, Seoul, Korea, where he is currently a Professor in the Department of Computer Science and Engineering.

His research interests include mobile wireless communication protocols and architectures, cognitive radio networking, and VANET. He has authored or co-authored over 113 papers in refereed international journals and conferences. He served as TPC member for IEEE INFOCOM 2008-2011, ACM MOBIHOC 2008-2009, IEEE ICCCN 2000-2008, and over 100 international conferences. He is a Senior Member of IEEE.



Ding-Zhu Du received his M.S. degree in 1982 from Institute of Applied Mathematics, Chinese Academy of Sciences, and his Ph.D. degree in 1985 from the University of California at Santa Barbara. He worked at Mathematical Sciences Research Institute, Berkeley in 1985 – 1986, at MIT in 1986 – 1987, and at Princeton University in 1990-1991. He was an associate-professor/professor at Department of Computer Science and Engineering, University of Minnesota in 1991-2005, a professor at City University of Hong Kong in 1998 – 1999, a research professor at Institute of Applied Mathematics, Chinese Academy of Sciences in 1987-2002, and a Program Director at National Science Foundation of USA in 2002-2005. Currently, he is a professor at Department of Computer Science, University of Texas at Dallas and a WCU professor at Korea University.

His research interests include design and analysis of algorithms for combinatorial optimization problems in communication networks and bioinformatics. He has published more than 160 journal papers and 10 written books. He is the editor-in-chief of "Journal of Combinatorial Optimization" and "Discrete Mathematics, Algorithms and Applications". He is also in editorial boards of more than 15 journals.