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Construction of Directional Virtual Backbones with Minimum Routing Cost in Wireless Networks

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***Abstract*—It is well-known that the application of directional antennas can help conserve bandwidth and energy consumption in wireless networks. Thus, to achieve efficiency in wireless networks, we study a special *virtual backbone* (VB) using directional antennas, requiring that from one node to any other node in the network, there exists at least one directional shortest path all of whose intermediate directions should belong to the VB, named as *M*inimum r*O*uting *C*ost *D*irectional VB (MOC-DVB). In addition, VB has been well studied in *Unit Disk Graph* (UDG). However, radio wave based communications in wireless networks may be interrupted by obstacles (e.g., buildings and mountains). Thus, in this paper, we model a network as a general directed graph. We prove that construction of a minimum MOC-DVB is an NP-hard problem in a general directed graph and in term of the size of MOC-DVB, there exists an unreachable lower bound of the polynomial-time selected MOC-DVB. Therefore, we propose a distributed approximation algorithm for constructing MOC-DVB with approximation ratio of $1 + \ln K + 2\ln \delta_D$, where K is the number of antennas on each node and δ_D is the maximum *direction degree* in the network. Extensive simulations demonstrate that our constructed MOC-DVB is much more efficient in the sense of MOC-DVB size and routing cost compared to other VBs.**

I. Introduction

In wireless networks, broadcasting and routing happen very frequently. To achieve efficient broadcasting, a backbone is constructed in a wired network. Only nodes selected in the backbone will help forward packets and spread the packets throughout the whole network. Thus, inspired by the physical backbone in wired networks, it is believed that a *virtual backbone* (VB) in a wireless network will help achieve efficient broadcasting. In addition, virtual backbones can help reduce routing path searching time and routing table size by constraining the searching space from the whole network to the selected virtual backbone.

In most virtual backbone research, *Connected Dominating Set* (CDS) is regarded as an efficient and practical option as a virtual backbone in wireless networks. Those researches model a network as $G = (V, E)$, where V represents all nodes in the network and E represents all bidirectional links in the network. A CDS of the network (denoted as S) is a subset of V, meeting two characteristics as follows: 1). $\forall v \in (V \backslash S)$, $\exists u \in S$ having $(u, v) \in E$. 2). the induced subgraph by S from G should be connected.

Most CDS researches focus on how to reduce the number of nodes selected to form a CDS. However, there are two drawbacks of these CDS researches. On one hand routing cost through CDS may increase a lot compared to the minimum routing cost in the network. Assume that every node has uniform transmission radius, hence, routing cost of a path can be evaluated by the hop count on the path (a.k.a the length of the path). Length of routing paths through CDS may increase a lot compared to the shortest path in the original network, since many shortest paths in the original graph are not included in CDS's induced subgraphs any more. For example, nodes A, E, F, and K construct a minimum CDS in Fig. 1 (a). Shortest path between H and J in the original network is $p(H, J) = \{H \leftrightarrow I \leftrightarrow J\}$ of length 2, however, the path between H and J through the minimum CDS is $p'(H, J) = \{H \leftrightarrow F \leftrightarrow A \leftrightarrow E \leftrightarrow J\}$ of length 4 — twice as that in the original network. Longer routing paths will consume much more energy and decrease packets' delivery ratio [1]. On the other hand, selected nodes in CDS may forward packets in unnecessary directions since a rather small portion of the transmission power is actually intercepted by the intended receivers. In Fig. 1, we divide every node's transmission range into four uniform directions. The ith direction of node A is denoted as d^i_A. We assume that the transmission energy cost is impacted directly by the angle of directional antennas and nodes can only receive the messages from the directions where they are in. For example, in Fig. 1 (a), H is only in d^3_F, it can only receive messages from F through d^3_F. Thus, for the minimum CDS in Fig. 1 (a), forwardings are unnecessary in $\{d^1_A, d^2_A, d^3_A, d^1_E, d^4_E\}$ since no receiver is in these directions. Thus, the power spreading in these directions cannot make efficient use of power and even worse, collisions may happen in the redundant directions. Meanwhile, selection is also unnecessary in $\{d^2_K, d^4_K\}$ even though there exist neighbors in these directions, because where d^2_K or d^4_K is needed, we can use other directions to replace them. For instance in Fig. 1 (a), we can switch off d^4_K and let G be dominated by d^3_F only.

[1] proposed a concept of *diameter*. *diameter* in a graph is defined as the length of the longest shortest path of the graph. Mohammed *et al.* [1] used this concept as a new metric to evaluate the quality of a CDS. If *diameter* of the induced subgraph of a CDS is small, then the CDS is regarded as a good construction since maximum hops of the routing path through the CDS will not be too large. However,

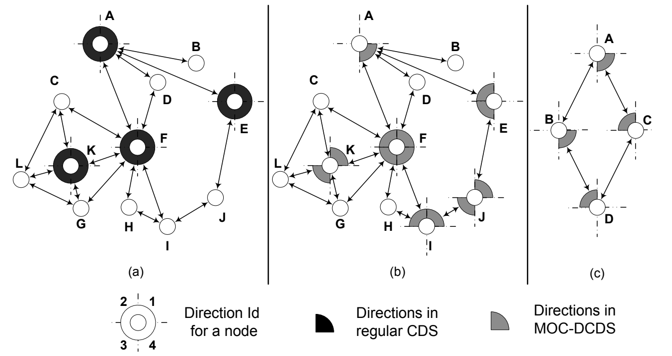


Fig. 1. Comparison between CDS and MOC-DVB

to better the worst case does not mean we can better the average performance. Thus, Kim *et al.* [2] proposed another concept *A*verage *B*ackbone *P*ath *L*ength (ABPL) which is used to evaluate the average routing path length through a CDS. However, both [1], [2] failed to consider shortest path constraint even though they noticed the necessity of reducing routing path through a CDS. Additionally, to save energy cost in unnecessary directions, [3] proposed *Directional* CDS (DCDS) with directional antennas. Different from CDS, DCDS aimed to select a subset of directions switched on to construct a DCDS. Compared to regular CDS, DCDS saves many unnecessary directions. However, routing path length is ignored in DCDS and running time of the construction of DCDS is significant.

Based on the previous discussion, to improve the performance of VB-based protocols (e.g., routing and broadcasting) in wireless networks, we will study a special VB named *M*inimum r*O*uting *C*ost *D*irectional VB (MOC-DVB) where directional antennas are exploited. The model of directional antennas used in this paper is directional forwarding and omni-reception. Since we assume that every node in a network shares the same transmission range, hence, minimum routing cost also means minimum routing hops. We allow that the source node of every broadcasting or routing can initiate new messages in any direction no matter whether the source node is in MOC-DVB or not. Hence, the MOC-DVB problem aims to find a subset of directions of a graph requiring that, for any pair of nodes in the graph, there exists at least one shortest path and all directions of the intermediate nodes on this shortest path must belong to MOC-DVB. A minimum MOC-DVB in a graph is the MOC-DVB with the smallest number of directions among all MOC-DVBs in the graph. For instance, in Fig. 1 (b), d_A^4, d_E^2, d_E^3, d_F^1, d_F^2 d_F^3, d_F^4, d_I^1, d_I^2, d_J^1, d_J^3, d_K^1, and d_K^3 construct a minimum MOC-DVB with 13 directions switched on where every routing path through the minimum MOC-DVB is also shortest path in the original graph. Hence, MOC-DVB can achieve energy-efficient VB-based broadcasting and routing.

In addition, we do not require that the selected directions induce a strongly connected VB, however, we allow that every source node can initiate a message in any direction. With this assumption, the message can be delivered to any other node in the network only through the directions in the MOC-DVB even though the induced subgraph of MOC-DVB is disconnected. For example, in Fig. 1 (c), d_A^4, d_B^4, d_C^2, and d_D^2 also construct a minimum MOC-DVB even though the induced subgraph by the four directions is not connected. However, routing or broadcasting of any message can be achieved by directions in this MOC-DVB. If B has a message for C, B will initiate the message in d_B^1, then A will forward it to C through d_A^4 — by path $\{B^1 \rightarrow A^4 \rightarrow C\}$. Similarly, if C has a message for B, the path $\{C^3 \rightarrow D^2 \rightarrow B\}$ will be used. $\{C^3 \rightarrow D^2 \rightarrow B\}$ means C initiate a message in its third direction, D receives the message in an omni-reception way and forwards the message to B in its second direction.

Our contributions in this paper are as follows:

1) To achieve efficient VB based routing and broadcasting, we propose a special VB — MOC-DVB. MOC-DVB aims to find a subset of directions of a graph. The selected directions construct a VB with shortest path constraints.
2) We prove that finding a minimum MOC-DVB is NP-hard in a general directed graph, and there exists an unreachable lower bound for selection of MOC-DVB in polynomial time which means that we cannot construct a MOC-DVB in polynomial time and the size of the selected MOC-DVB is below the lower bound we prove.
3) We prove an upper bound for selection of MOC-DVB in polynomial time. One distributed approximation algorithm is proposed to construct MOC-DVB and the size of the selected MOC-DVB is indeed under the upper bound we prove.

The rest of the paper will be organized as follows: in Section II, we will review the related work on VB. In Section III, we will introduce the communication model and the formal definition of MOC-DVB. An equivalent problem (named 2hop-DVB) to MOC-DVB will be introduced. We will prove that finding a minimum MOC-DVB is an NP-hard problem, by proving that finding a minimum 2hop-DVB is NP-hard. In this section, we will also prove that there exists an unreachable lower bound of MOC-DVB. In Section IV, we will introduce a distributed approximation algorithm for selecting a MOC-DVB. In Section V, the approximation ratio of the distributed algorithm will be proved. In Section VI, extensive simulations will demonstrate that the maximum and the average routing cost through MOC-DVB are reduced significantly compared to that through CDSs. Finally, the paper will be concluded in Section VII.

II. Related Work

The research work on selecting a VB has never been interrupted because of its remarkable contributions to wireless networks. Nearly all research work on VB focuses on how to construct a CDS. It has been proved that selection of a minimum CDS in a general graph is an NP-hard problem [4]. It has even been proved that the selection of a minimum CDS in a unit disk graph is an NP-hard problem [5]. To achieve an efficient backbone, directional CDS was studied using directional antennas [3].

A. Connected Dominating Set

We can categorize CDS selection algorithms into two types — one is 2-stage and the other one is 1-stage.

2-stage algorithms can also be divided into two types. The main idea of the first type is to select a *dominating set* (DS) and then add more nodes to the DS to make it connected. After the two steps, a CDS is selected. In contrast, the main idea of the other type is to construct a CDS with many more redundant nodes firstly. Then prune the redundant nodes from the selected CDS to construct a smaller CDS. In [6], two algorithms are proposed. One of the algorithms belongs to the first type of 2-stage CDS with an approximation ratio of $H(\delta) + 2$ where δ is the maximum node degree in the network and H is harmonic function. Butenko *et al.* [7] proposed a Leader algorithm belonging to the first type of CDS to select a CDS with size smaller than $8|OPT|+1$, where OPT represents the size of a minimum CDS. Recently, *Min et al.* [8] proposed a 2-stage algorithm based on the two technologies — Independent Set [9] and Steiner Tree [10]. The size of their selected CDS is smaller than $3.8|OPT| + 1.2$. The algorithm proposed in [11] belongs to the second type of a 2-stage algorithm. They achieved an approximation ratio of $O(n)$, where n is the number of nodes in the network.

The main idea of 1-stage algorithms aims to select a CDS directly, skipping the step of selection of a DS or a redundant CDS. Also in [6], one 1-stage algorithm was proposed yielding approximation ratio of $2H(\delta)+2$. Later, Ruan *et al.* [12] made a modification of the selection standard of DS. Therefore, the 2-stage algorithm in [6] is reduced to a 1-stage algorithm with approximation ratio of $3 + \ln \delta$.

B. Directional Connected Dominating Set

To achieve efficient VB based routing and broadcasting, directional antennas were used in some VB research. In DCDS [3], research aims to select as few directions as possible to construct a virtual backbone. The minimum DCDS problem was proven NP-complete [3]. They proposed a localized heuristic algorithm for constructing a DCDS. However, the time complexity in this paper is exponential under some circumstances since they need to compute all paths between any two nodes to make a decision whether one direction is selected or not in the worst case.

In addition, besides routing and broadcasting, virtual backbone has many other applications (e.g., topology control) in wireless networks. In this paper, we mainly focus on studying a VB to yield efficient VB-based broadcasting and routing.

III. Problem Statement

In this section, we will first introduce directional antennas that will be used in this paper. Then, we will introduce network model and MOC-DVB will also be defined formally. Moreover, to solve the problem of MOC-DVB, an equivalent problem named 2hop-DVB will be defined and it is proved that finding a minimum MOC-DVB or 2hop-DVB is NP-hard in a general directed graph.

A. Directional Antenna

[13] illustrates the techniques used in smart antenna systems to form directional transmission and/or reception beams. Similar to the directional antennas used in [14], in this paper, we assume that the directional antennas used in one network are regular, aligned and nonoverlapped. For the same network, uniform directional antennas are used (e.g., angles and shapes). Thus, nodes' transmission areas are divided into several identical sectors. We can choose to send packets in demanded directions with the technique of switched beam, instead of in the whole transmission areas. For example, in Fig. 2 (b), four uniform directional antennas with 90^o angle are put on each node. We denote a node u's direction i as d_u^i while $d(x, y)$ is used to denote the direction of x where y is. Thus, d_A^1 in Fig. 2 (b) represents the first direction of A and $d(A, C) = 3$ represents C is in the third direction of A. There are two kinds of reception — one is omni-reception which means nodes can receive messages from its neighbors in any direction, and the other one is directional reception which means nodes can only receive messages from neighbors in predetermined directions. In the rest part of the paper, we adopt omni-reception. There is a directional link from x to y only when y is in x's transmission range and x switches on the direction where y is, denoted as $x^{d(x,y)} \to y$ representing that x can initiate a new message or forward a message to y in the direction $d(x, y)$. In Fig. 2 (b), C is in the transmission area of d_A^3, thus, there is a directional link from A to C denoted as $A^{d(A,C)} \to C$, where $d(A, C) = 3$. Therefore, we have a directed edge $A^3 \to C$ in Fig. 2 (c). If C turns off its d_C^1 where A is while A turns on its d_A^3, then we only have the link $A^3 \to C$ but $C^1 \to A$. In this paper, one node can only appear in at most one direction of each other node and one node can only receive messages from the direction where it is. Hence, in Fig. 2 (b), C can only receive message from A sent in d_A^3.

In addition, we assume that the transmission energy cost is impacted directly by the angle of directional antennas (θ) and transmission radius (r). In the rest part of the paper, given the same r, the energy cost of each directional transmission is $(\theta/360)$ that of an omni-directional transmission. Energy cost of a directional antennas of 90^o is twice as that of a directional antennas of 45^o.

B. Network Model

In this paper, we assume that every node shares the same transmission range. However, it does not mean that two nodes can communicate with each other when they are in each other's switched on transmission direction due to the existence of obstacles [15]. Reichenbach *et al.* [15] find obstacles may cause diffraction, scattering, blocking, and reflection. In Fig. 2 (a), all of the four results of an obstacle will circumvent the successful radio wave transmission between nodes X and Y with spatial positions close enough to each other.

Considering the existence of obstacles and the use of directional antennas, it is reasonable to model a network as a general directed graph $G = (V, D, E)$. V represents the set of

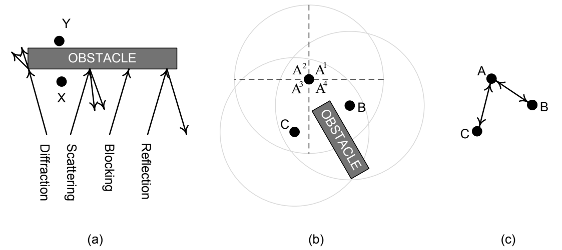


Fig. 2. Obstacle and Network Model

nodes in the network. D represents all nodes' directions in the network. E is a directed edge set representing all directional links in the network. In this paper, G is a strongly connected general directed graph. $N(v)$ represents all neighbors of node v and $N(d_v^i)$ represents the neighbor of v in the direction d_v^i. In Fig. 2(c), $N(d_A^1) = N(d_A^2) = \phi$, $N(d_A^3) = \{C\}$, $N(d_A^4) = \{B\}$, and $N(A) = \{B, C\}$. We define *direction degree* as the number of neighbors in one direction. In Fig. 2 (b), nodes A, B, and C are in each other's transmission range. There is an obstacle between B and C while there is no obstacle between A and C, or A and B. Thus, the network is modeled as a general directed graph G in Fig. 2 (c), where $V = \{A, B, C\}$, $D = \{d_A^1, d_A^2, d_A^3, d_A^4, d_B^1, d_B^2, d_B^3, d_B^4, d_C^1, d_C^2, d_C^3, d_C^4\}$ and $E = \{A^3 \to C, A^4 \to B, C^1 \to A, B^2 \to A\}$.

Therefore, to achieve efficient VB-based broadcasting, we need to reduce the number of selected directions for forwarding. To achieve energy efficient VB-based routing and broadcasting, we study a special VB — for any one node u to any other node v, there exists at least one shortest path all of whose intermediate directions belong to the VB.

C. Problem Definition

In this paper, we define the shortest paths from any node u to another node v as the directional path with the smallest hops among all paths from u to v, denoted as $p(u \to v)$. We use $P(u \to v)$ to denote all shortest paths from u to v. For one directional path, the directions of intermediate nodes used on this path are defined as intermediate directions. We denote a directional shortest path from u to v as $p(u \to v) = \{u^{d(u,w_1)} \to w_1^{d(w_1,w_2)} \to ... \to w_i^{d(w_i,w_{i+1})} \to w_{i+1}^{d(w_{i+1},w_{i+2})} \to ... \to w_k^{d(w_k,v)} \to v\}$ representing u initiates a message in the direction $d(u, w_1)$ where w_1 is, then w_1 receives the message using omni-reception and forwards it to w_2 using direction $d(w_1, w_2)$. The message will go through w_2, ... , w_k, and lastly, destination v can receive the message using omni-reception from w_k. For example in Fig. 1 (a), there exist three shortest paths from A To F — $p_1(F \to L) = \{F^2 \to C^3 \to L\}$, $p_2(F \to L) = \{F^3 \to K^3 \to L\}$, and $p_3(F \to L) = \{F^3 \to G^2 \to L\}$. $P(F \to L) = \{p_1, p_2, p_3\}$. In this paper, we define the distance from u to v as the hop count on the shortest path from u to v, known as hops distance [16] and denoted as $Dist(u \to v)$. In Fig.1 (a), $Dist(F \to L) = 2$. Based on the fact that the energy cost of every one-hop transmission in one direction of a network is predetermined, saving routing energy cost is equivalent to reducing routing hops that also means reducing intermediate directions.

To achieve efficient broadcasting and routing, we propose MOC-DVB which is formally defined in Def. 1.

Definition 1 (MOC-DVB). *The* Minimum rOuting Cost Directional Virtual Backbone *problem (MOC-DVB) is to find a direction set $Sub\_D \subseteq D$ in $G = (V, D, E)$ such that*

1) *Based on Sub_D, we have Sub_V which is a node set and $\forall u \in Sub\_V, \exists\ d_u^i \in Sub\_D$. Meanwhile, $\forall v \in V \backslash Sub\_V$, $\exists u \in Sub\_V$ and $d_u^i \in Sub\_D$ having v in ith directional transmission area of u. That is $\exists (u^i \to v) \in E$.*
2) *$\forall\ u, v \in V$, if $Dist(u \to v) > 1$, $\exists\ p_i(u \to v) \in P(u \to v)$, all intermediate directions on $p_i(u \to v)$ belong to MOC-DVB.*

Our definition is for the graphs which are not complete graphs. For the special case of a complete graph, pick a node randomly and select its directions which have neighbors to construct a DVB. To simplify the construction of a MOC-DVB, we find an equivalent VB to MOC-DVB, named 2hop-DVB. We prove that the two types of VB are equivalent to each other in Lemma 1.

D. 2hop-DVB

Definition 2 (2hop-DVB). *The* 2hop Directional Virtual Backbone *problem (2hop-DVB) is to find a direction set $(Sub\_D)' \subseteq D$ in $G = (V, E, D)$ such that*

1) *Based on $(Sub\_D)'$, we have $(Sub\_V)'$ which is a node set and $\forall u \in (Sub\_V)', \exists\ d_u^i \in (Sub\_D)'$. Meanwhile, $\forall v \in V \backslash (Sub\_V)'$, $\exists u \in (Sub\_V)'$ and $d_u^i \in (Sub\_D)'$ having v in ith directional transmission area of u. That is $\exists (u^i \to v) \in E$.*
2) *$\forall\ u, v \in V$, if $Dist(u \to v) = 2$, $\exists\ p_i(u \to v) \in P(u \to v)$, all intermediate directions on $p_i(u \to v)$ belong to 2hop-DVB.*

No matter in MOC-DVB or in 2hop-DVB, source nodes which initiate new packets can send the new packets in any direction. For those source nodes belong to Sub_V or $(Sub\_V)'$, they can initiate new packets in the directions which do not belong to Sub_D. However, we require all intermediate directions belong to Sub_D or $(Sub\_D)'$.

Lemma 1. *A direction set Sub_D is a MOC-DVB if and only if it is also a 2hop-DVB.*

Proof: It is trivial to get the conclusion that a MOC-DVB is also a 2hop-DVB based on the Def. 1 and Def. 2. From the definitions, we know that a MOC-DVB is also a 2hop-DVB because MOC-DVB has constraint on all pair of nodes with distance bigger than 1 while 2hop-DVB has constraints on all pairs of nodes with distance of exactly 2.

To prove the equivalence between 2hop-DVB and MOC-DVB, the key point here is whether a 2hop-DVB is also a MOC-DVB. Next, we will prove that a 2hop-DVB also meets the two constraints in Def. 1.

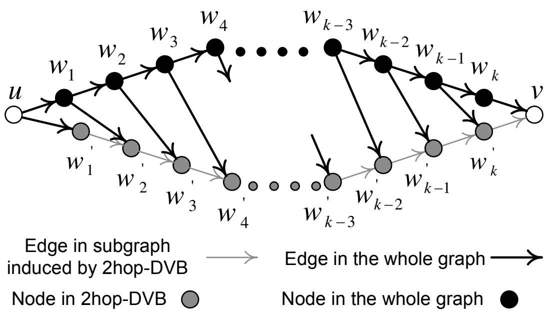


Fig. 3. Equivalence between MOC-DVB and 2hop-DVB

Because of the first constraint of 2hop-DVB in Def. 2, a 2hop-DVB must meet the first constraint of MOC-DVB. To prove that a 2hop-DVB also meets the second constraint in MOC-DVB, we need to prove that for any pair of nodes u and v, having $Dist(u \to v) > 1$, there exists one shortest path $p(u \to v)$ and all intermediate directions used on $p(u \to v)$ belong to the 2hop-DVB. Assume $p(u \to v) = \{u^{d(u,w_1)} \to w_1^{d(w_1,w_2)} \to w_2^{d(w_2,w_3)} \to ... \to w_k^{d(w_k,v)} \to v\}$ in the original graph (in Fig. 3), then we can get $Dist(w_{k-1} \to v) = 2$. According to the definition of 2hop-DVB, we can find a node $w_k^{'}$ and one of its selected directions $(w_k^{'})^{d(w_k^{'},v)}$ in the 2hop-DVB to form a replacement path $p^{'}(u \to v) = \{u^{d(u,w_1)} \to w_1^{d(w_1,w_2)} \to w_2^{d(w_2,w_3)} \to ... \to w_{k-2}^{d(w_{k-2},w_{k-1})} \to w_{k-1}^{d(w_{k-1},w_k^{'})} \to (w_k^{'})^{d(w_k^{'},v)} \to v\}$. Similarly, we can find directions $(w_{k-1}^{'})^{d(w_{k-1}^{'},w_k^{'})}$, ... , $(w_j^{'})^{d(w_j^{'},w_{j+1}^{'})}$, ... , $(w_2^{'})^{d(w_2^{'},w_3^{'})}$, $(w_1^{'})^{d(w_1^{'},w_2^{'})}$ in the 2hop-DVB to construct a replacement directional path from u to v $p^{final}(u \to v) = \{u^{d(u \to w_1^{'})} \to (w_1^{'})^{d(w_1^{'},w_2^{'})} \to (w_2^{'})^{d(w_2^{'},w_3^{'})} \to ... \to (w_j^{'})^{d(w_j^{'},w_{j+1}^{'})} \to ... \to (w_{k-1}^{'})^{d(w_{k-1}^{'},w_k^{'})} \to (w_k^{'})^{d(w_k^{'},v)} \to v\}$, where all intermediate directions belong to the 2hop-DVB. Meanwhile, the final replacement path $p^{final}(u \to v)$ has the same number of hops as that of $p(u \to v)$. Hence, we can conclude that a 2hop-DVB also meets the second constraint of MOC-DVB's definition. Therefore, a 2hop-DVB is also a MOC-DVB.

In sum, MOC-DVB and 2hop-DVB are equivalent to each other. That is, $(Sub\_D)' = Sub\_D$ and $(Sub\_V)' = Sub\_V$. ■

If a special case of one problem is proven NP-hard then the problem must be regarded as an NP-hard problem. We discuss the special case of 2hop-DVB that all nodes in the network have uniform directional antennas of 360^o, denoted as 2hop-DVB-360^o. Hence, a node can be selected in Sub_V when and only when its sole direction of 360^o is selected in Sub_D. That is, in 2hop-DVB-360^o, $|V| = |D|$ and $|Sub\_V| = |Sub\_D|$. Finding a minimum 2hop-DVB-360^o in a general directed graph can be proven NP-hard by reduction from 2hop-CDS to 2hop-DVB-360^o. 2hop-CDS has been studied in [17] and it has been proven that finding a minimum 2hop-CDS in a general graph is NP-hard. Before we prove finding a minimum 2hop-DVB-360^o is NP-hard, we first recall the definition of 2hop-CDS as given in Def. 3.

Definition 3 (2hop-CDS)**.** *Given a general bidirected graph $G_{bi} = (V_{bi}, E_{bi})$, the* 2-hop Shortest Path Connected Dominating Set *problem (2hop-CDS) is to find a minimum-size node set $S_{bi} \subseteq V_{bi}$ such that*

1) *$\forall$ $v_{bi} \in V_{bi} \backslash S_{bi}$, $\exists$ $u_{bi} \in S_{bi}$, such that $(v_{bi}, u_{bi}) \in E_{bi}$.*
2) *The induced graph $G_{bi}[S_{bi}]$ is connected.*
3) *$\forall$ $u_{bi}, v_{bi} \in V_{bi}$, if $H(u_{bi}, v_{bi}) = 2$, then $\exists$ $p(u_{bi}, v_{bi})$, $p(u_{bi}, v_{bi}) \backslash \{u_{bi}, v_{bi}\} \subseteq S_{bi}$, where $H(u_{bi}, v_{bi})$ represents the hop distance between u_{bi} and v_{bi}, $p(u_{bi}, v_{bi})$ represents one shortest path between u_{bi} and v_{bi}.*

Lemma 2. *Selecting a minimum* 2hop-DVB-360^o *in a general directed graph is NP-hard.*

Proof: We first show that 2hop-DVB-$360^o \in NP$. Given a graph $G = (V, D, E)$ and an integer k. The certification we choose is the 2hop-DVB-360^o $Sub\_V \subseteq V$ and $Sub\_D \subseteq D$. The verification algorithm affirms that $|Sub\_V| = k$, $|Sub\_D| = k$ and then it checks, for each pair of nodes $u, v \in V$ having $Dist(u \to v) = 2$, $\exists w \in Sub\_V$ *and* $d(w, v) \in Sub\_D$ to form a directional path $\{u^{d(u,w)} \to w^{d(w,v)} \to v\}$. This verification can be performed straightforwardly in $O(n^3)$ — polynomial time.

Next, we prove that finding a minimum 2hop-DVB-360^o is NP-hard in a directed graph by showing that 2hop-CDS $\leq_P$ 2hop-DVB-360^o.

$\forall v_{bi} \in G_{bi}$, we deploy a 360^o directional antenna on v_{bi}. This will not change the topology of G_{bi}. Thus we get a general directed graph $G = (V, D, E)$, where $V = V_{bi}$, $|D| = |V|$, and E represents the directed edges in G. $\forall v_{bi} \in G_{bi}$, it will be denoted as v in G. When there is an edge $(u_{bi} \leftrightarrow v_{bi})$ between u_{bi} and v_{bi} in G_{bi}, then there must exist two directed edges $(u^{d(u,v)} \to v)$ and $(v^{d(v,u)} \to u)$ in G, where $d(v, u)$ is the sole direction of v and $d(u, v)$ is the sole direction of u. For two nodes u and v in G, we can derive $Dist(u \to v) = 2$ in G from $H(u_{bi}, v_{bi}) = 2$ in G_{bi}. Meanwhile, we can derive $H(u_{bi}, v_{bi}) = 2$ in G_{bi} from $Dist(u \to v) = 2$ in G.

We claim that G_{bi} has a 2hop-CDS solution S_{bi} of size at most k satisfying Def. 3 if and only if G has a 2hop-CDS-360^o of size at most k satisfying Def. 2.

(1). We first prove when $|S_{bi}| \leq k$, then we can obtain a Sub_D in G having $|Sub\_D| \leq k$. Our claim holds trivially. If 2hop-CDS has a solution of S_{bi}, then all corresponding 360^o directions of nodes in S_{bi} construct a direction set which meets the two requirements in Def. 2 and $Sub\_V = S_{bi}$. We will prove item by item. In 2hop-CDS, any node v_{bi} outside S_{bi} will have an adjacent node u_{bi} in S_{bi} and edge $(u_{bi}, v_{bi}) \in G_{bi}$. Thus, for any node v outside Sub_V, it will have an adjacent node u in Sub_V with its sole 360^o direction switched on and v is in the direction. There must be a directed edge $(u^{d(u,v)} \to v) \in G$ meeting the first constraint in Def. 2. On the other hand, for any pair of nodes u and v in G_{bi} having distance $H(u, v) = 2$, there exists at least one shortest path $p(u, v) = \{u \leftrightarrow w \leftrightarrow v\}$, where $w \in S_{bi}$. Thus, in 2hop-DVB-360^o, from node $u \in G$ to node $v \in G$ having $Dist(u \to v) = 2$, there exists at least one directional path $p(u \to v) = \{u^{d(u,w)} \to w^{d(w,v)} \to v\}$, where $d(u,w)$ and $d(w,v)$ are the sole directions of u and w respectively. Since node w belong to 2hop-CDS, its sole direction must be in Sub_D. Thus, $p(u \to v)$'s intermediate direction $d(w,v)$ of w must belong to 2hop-DVB-360^o. This meets the second requirement in Def. 2.

(2). Conversely, suppose that G has a 2hop-DVB-360^o Sub_D of size at most k, then its corresponding 2hop-CDS has S_{bi} of size at most k. Given a 2hop-DVB-360^o solution Sub_D, then all nodes in Sub_V should construct a solution S_{bi}, that is, $Sub\_V = S_{bi}$. Similarly, we need to prove that S_{bi} meets the three requirements in Def. 3. Firstly, in 2hop-DVB-360^o, every node v outside Sub_V has an adjacent node u in Sub_V having a direction $d(u,v) \in Sub\_D$ and the directed edge $(u^{d(u,v)} \to v) \in E$. Hence, we can get that there must exist a bidirected edge $(u \leftrightarrow v) \in E_{bi}$. Thus, it is obvious that every node v_{bi} outside S_{bi} in G_{bi} will have an adjacent node $u_{bi} \in S_{bi}$ and $(u_{bi} \leftrightarrow v_{bi}) \in E_{bi}$. The first item is proved. Secondly, we need to prove that S_{bi} induces a connected subgraph in G_{bi}. If we assume that the induced subgraph by S_{bi} is not connected, there exists a contradiction. If the subgraph is disconnected, there should be several components (C_1, C_2, ... , C_m) in the subgraph. Nodes within one component can communicate while nodes in different components cannot communicate in the subgraph. Find two nodes $x_{bi} \in C_1$ and $y_{bi} \in C_2$ whose distance is smallest among all pairs of nodes whose distances are bigger than 1, having one in C_1 and the other one in C_2 respectively. This means there is no shortest path $p(x_{bi}, y_{bi})$ with all intermediate nodes belonging to S_{bi}. As a result, in 2hop-DVB-360^o, no matter in which direction x initiates a message to the destination y, the message cannot be delivered by directions in Sub_D. That is, Sub_D cannot form a 2hop-DVB-360^o. The contradiction happens. Thus, the induced subgraph by S_{bi} is connected. In 2hop-DVB-360^o, for one node u and any other node v, there exists a shortest path $p(u \to v) = \{u^{d(u,w)} \to w^{d(w,v)} \to v\}$ and the intermediate direction $d(w,v)$ belongs to 2hop-DVB-360^o. Due to the fact that deployment of antennas does not change G_{bi}'s topology and its characteristic of "bidirectional", for u_{bi} and v_{bi} in G_{bi}, there exists at least one shortest path $p(u_{bi}, v_{bi}) = \{u_{bi} \leftrightarrow w_{bi} \leftrightarrow v_{bi}\}$. The third item in Def. 3 is proven. Thus, if G has a 2hop-DVB-360^o of size at most k, then G must have a 2hop-DVB-360^o of size k at most.

In sum, finding a minimum 2hop-DVB-360^o is NP-hard in a general directed graph.

■

Based on Lemma 2, we get the Theorem 1 trivially.

Theorem 1. *Selecting a minimum MOC-DVB or* 2*hop-DVB is NP-hard in a general directed graph.*

IV. ALGORITHM

Before introducing the details of algorithm, we first introduce how to collect neighbor information. Every node v in a network will be assigned a unique node id $id(v)$ and every direction i of v is assigned a unique direction id $id(d_v^i)$.

A. Neighbor Information Maintenance

3-round "hello" messages are used to collect neighbor information. We use K to denote the numbers of directional antennas deployed on each node. In each round, every node will send "Hello" messages K times in K directions. "Hello" message is piggybacked with the sender v's $id(v)$, direction id $id(d_v^i)$. Once a node u receives the "Hello" message piggybacked with $id(v)$ and $id(d_v^i)$, u knows that there is a direction link $v^i \to u$. After the first round "Hello" messages, every node v will know all directional links ended at v. In the second round, besides sender id and direction id, every "Hello" message will also be piggybacked the sender's all 1-hop directional links collected in the first round. By collecting the second round "Hello" messages, every node can collect all directional links started or ended at it. For each link $v^i \to u$, v will add u to $N(d_v^i)$. Lastly, the third round "Hello" messages will be used. In the third round, every node will send out "Hello" messages piggybacked all information collected in the second round. Lastly, every node v will know all directional links from any node to any other node which is within 2 hops away from it.

B. Distributed Algorithm

Algorithm 1 Distributed Selection of 2hop-DVB

Step 1. Each node v with nonempty $W(d_v^i)$, $\forall i \in \{1,2,...,K\}$, calculates $f(d_v^i) = |W(d_v^i)|$. It picks out the direction d_v^m with the maximum f value, stored as $f(v)$. If there are more than one such direction, it breaks tie by choosing the one with the lowest direction id. It sends $f(v)$ to all its neighbors;

Step 2. Each node v computes maximum f value among received $f(u)$'s from its neighbors in Step 1 (including itself). It sends a flag to the node u having the maximum f value. If there are more than one such u, then it breaks tie by choosing the one with lowest node id;

Step 3. If a node v receives flags from all its neighbors, it adds d_v^m to Sub_D and adds v to Sub_V. Then it sends $W(d_v^m)$ to all its neighbor and sets $W(d_v^m) = \phi$;

Step 4. If a node u receives $W(d_v^m)$ from v, it passes $W(d_v^m)$ to all neighbors;

Step 5. If a node u receives $W(d_v^m)$, it computes union U of such $W(d_v^m)'s$ and updates W of all directions of u by setting $W(d_u^i) \leftarrow W(d_u^i) - U$, $\forall i \in \{1,2,...,K\}$.

The basic idea of the algorithm introduced in this paper is a greedy strategy. Before introducing the algorithm, we first clarify some definitions which will be used in the algorithms. For each direction d_v^i, it will store $W(d_v^i)$. Initially, $W(d_v^i) = \{(u \to w) \mid Dist(u \to w) = 2 \ and \ \exists \ p(u \to w) = \{u^{d(u,v)} \to v^{d(v,w)} \to w\}, \ where \ w \ is \ in \ d_v^i\}$. At each step, we choose the direction which has the maximum $|W(d_v^i)|$.

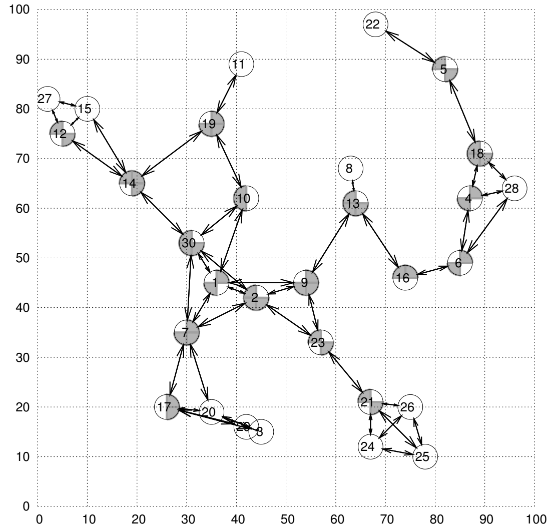


Fig. 4. An example of MOC-DVB by Alg. 1

The algorithms will stop when $W(d_v^i) = \phi$ for all nodes in all directions. The details are given in Alg. 1.

Fig. 4 shows an example in a $100m \times 100m$ area. 30 nodes are deployed in the area. All the 30 nodes share the same transmission range $20m$ and uniform directional antennas of 90^o are deployed on the 30 nodes. Grey sectors, as shown in the figure, represent a MOC-DVB obtained by Alg. 1. In the resultant graph, there are 44 directions in the selected MOC-DVB. For example, $W(d_{12}^2) = W(d_{15}^2)$. With the help of node id, the second direction of 12 is selected firstly to MOC-DVB. Then the set W of direction d_{15}^2 will be recalculated as ϕ. Hence, the second direction of 15 will not be selected.

V. Theoretical Analysis

In this section, we will prove the correctness and approximation ratio of our distributed algorithm. We also prove that we cannot find a polynomial time algorithm to construct a MOC-DVB achieving approximation ratio of $\rho \ln \delta_D$ unless $NP \subseteq DTIME(n^{O(\log \log n)})$, where ρ is an arbitrary positive number ($\rho < 1$) and δ_D is the maximum *direction degree* in a network. Firstly, we will prove that our algorithm constructs a MOC-DVB.

Theorem 2. *The subset of directions selected in Alg. 1 is both a* 2*hop-DVB and a MOC-DVB.*

Proof: Firstly, we prove that the selected directions meet the first constraint by contradiction. We assume that there is one node v outside the selected Sub_V by Alg. 1, is not in any direction in Sub_D. Then, there must exist a node u with $Dist(u \to v) = 2$ and directional paths from u to v — $\{u^{d(u,w_1)} \to w_1^{d(w_1,v)} \to v\}$, ... ,$\{u^{d(u,w_k)} \to w_k^{d(w_k,v)} \to v\}$. As a result, $W(d(w_i, v)) \neq \phi$, $\forall i \in [1, k]$. Then the algorithm should not stop. Contradiction happens. Thus, every node outside Sub_V must be in at least one direction in Sub_D, meeting the first constraint.

Secondly, we will prove by constradication that Sub_D selected by Alg. 1 meets the second constraint in Def. 2. If Sub_D does not meet the second constraint, then there must exists one pair of nodes u and v with $Dist(u \to v) = 2$ and several directional paths from u to v — $\{u^{d(u,w_1)} \to w_1^{d(w_1,v)} \to v\}$,...,$\{u^{d(u,w_k)} \to w_k^{d(w_k,v)} \to v\}$. Similarly, $W(d(w_i, v)) \neq \phi$, $\forall i \in [1, k]$. Then the algorithm should not stop. Contradiction happens. Thus, Sub_D meets the second constraint in Def. 2.

In sum, the selected directions by Alg. 1 construct a 2hop-DVB. Since 2hop-DVB is equivalent to MOC-DVB, it is also a MOC-DVB. ■

In [17], Ding *et al.* proved that for 2hop-CDS, there does not exist a polynomial time algorithm having approximation ratio of $\rho \ln \delta$, where $\forall \rho < 1$ and δ is the maximum node degree in a graph, unless $NP \subseteq DTIME(n^{O(\log \log n)})$. Based on this conclusion, we show that there exists an unreachable approximation ratio of any polynomial time algorithm for constructing a 2hop-DVB.

Theorem 3. *Neither MOC-DVB nor* 2*hop-DVB has a polynomial time algorithm with approximation ratio $\rho \ln \delta_D$, where $\forall \rho < 1$ and δ_D is the maximum* direction degree *in the input graph, unless $NP \subseteq DTIME(n^{O(\log \log n)})$.*

Proof: Based on the proof of Lemma 2, an immediate corollary of our claim is that optimal 2hop-CDS of a graph G_{bi} has size $opt_{2hop-CDS}$ if and only if optimal 2hop-DVB-360^o of the corresponding graph G has size of $opt_{2hop-DVB-360^o}$, where $opt_{2hop-CDS} = opt_{2hop-DVB-360^o}$. We use contradiction method to prove that we cannot propose a polynomial time algorithm to construct a 2hop-DVB-360^o with approximation ratio of $\rho \ln \delta_D$.

Assume G has a polynomial time solution D for 2hop-DVB-360^o with size at most $(\rho \ln \delta_D)(opt_{2hop-DVB-360^o})$ for some constant $\rho < 1$. Thus, we can find a polynomial time solution to 2hop-CDS with size at most $(\rho \ln \delta)(opt_{2hop-CDS})$. This implies that $NP \subseteq DTIME(n^{O(\log \log n)})$. Therefore, the assumption that G has a polynomial time solutiion with size at most $(\rho \ln \delta_D)(opt_{2hop-DVB-360^o})$ for some constant $\rho < 1$ is incorrect. In sum, Theorem 3 is proved. ■

Definition 4 (Hitting Set)**.** *Given a finite set A and a collection $\mathcal{Y}$ such that $\forall Y \in \mathcal{Y}$ having $A \bigcap Y \neq \emptyset$, Hitting Set is a subset $R \subseteq A$ such that $\forall Y \in \mathcal{Y}$ having $R \bigcap Y \neq \emptyset$.*

Theorem 4. *A polynomial time approximation algorithm can be designed for* 2*hop-DVB of a graph $G = (V, D, E)$ with performance ratio of $1 + \ln K + 2 \ln \delta_D$ at most, where δ_D is the maximum* direction degree *of the input graph and K represents the number antennas deployed on each node.*

Proof: For each pair of nodes u and v with $Dist(u \to v) = 2$, define $m(u \to v) = \{d(w, v) | (u^{d(u,w)} \to w^{d(w,v)} \to v)\}$. Now, finding a 2hop-DVB becomes finding a hitting set [18], where $A = D$ and $\mathcal{Y} = \{m(u \to v) | u, v \in V\}$. That means if we can find a solution to the hitting set problem, the solution is also a 2hop-DVB. In [18], the author proposed a greedy algorithm for finding a hitting set with performance ratio of $1 + \ln \gamma$ at most, where γ is the maximum number of Ys that an element can appear. In addition, by reducing 2hop-DVB to hitting set problem, we have $\gamma \leq (K-1)\delta_D^2 + \delta_D * (\delta_D - 1) \leq K * \delta_D^2$. Therefore, there exists a polynomial-time approximation algorithm for selecting a 2hop-DVB with performance ratio of $1 + \ln K + 2\ln \delta_D$. ■

Next, we will prove our distributed algorithm has the same bound by Theorem 5. We first give the definition of Set-Cover.

Definition 5 (Set-Cover)**.** *Given a collection $\mathcal{C}$ of subsets of a finite set X such that $\bigcup_{A\in\mathcal{C}} A = X$, find a minimum subcollection $\mathcal{A} \subseteq \mathcal{C}$ such that $\bigcup_{A\in\mathcal{A}} A = X$.*

Theorem 5. *Alg. 1 outputs the Sub_D with performance ratio $1 + \ln K + 2\ln \delta_D$, where δ_D is the maximum* direction degree *of the input graph and K represents the number antennas deployed on each node.*

Proof: Let $W_0(d)$ be the initial $W(d)$ and $X = \bigcup_{d\in D} W_0(d)$. Then problem of Def. 2 is equivalent to Set-Cover problem with base set X and collection $\mathcal{C} = \{W_0(d) | d \in D\}$. Suppose D^* gives the minimum solution $\{W_0(d) | d \in D^*\}$ to the Set-Cover problem of X and $\mathcal{C}$. We partition X into subsets $X(d)$ for $d \in D^*$ such that $\forall X(d)$, $X(d) \subseteq W_0(d)$.

Consider $d \in D^*$. Denote $f_0 = X(d)$ before the first round, where $f_0 \leq K * \delta_D^2$. We will make a charge to $(u \to w) \in X(v)$ when $(u \to w)$ is removed from $W(d)$ during the computation of distributed algorithm. When d is selected, we charge $1/f(d)$ to $\forall (w \to y) \in W(d)$.

Suppose that at the end of Step. 5 in Alg. 1 in the first round, $f_0 - f_1$ elements of $X(d)$ are charged. Then $\forall (u \to w) \in X(d)$ is charged by the value at most $1/f_0$. The total charge for those $f_0 - f_1$ removed elements is at most $(f_0 - f_1)/f_0$.

Similarly, let f_i be the number of uncharged elements in $X(d)$ at the end of Step 5 in the ith round. Then the total charge to elements of $X(d)$ is at most $(f_{i-1} - f_i)/f_i$.

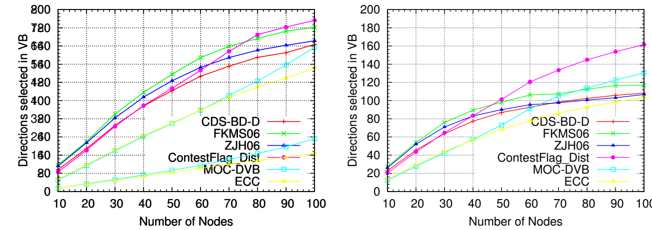
Suppose $f_k = 0$. Then all elements of $X(d)$ are charged at total value as follows:

$$\begin{aligned} \sum_{i=0}^{k-1} \frac{f_i - f_{i+1}}{f_i} \leq \sum_{i=1}^{f_0} \frac{1}{i} \quad &\leq \quad 1 + \int_1^{f_0} (1/x)dx \\ &\leq \quad 1 + \ln K + 2\ln \delta_D \end{aligned}$$

Note that when a direction d is selected, the total value of charging to $W(d)$ is 1. Therefore, the total value charging to elements of X is exactly the number of selected directions at the end of distributed algorithm. This number is bounded by $(1 + \ln K + 2\ln \delta_D) \times |D^*|$. ■

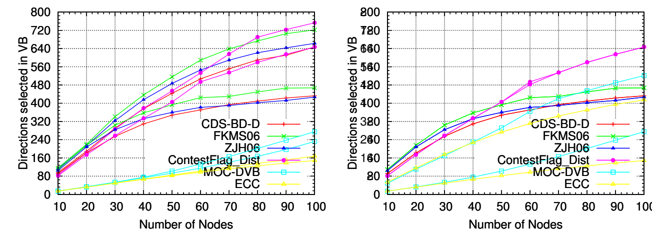
VI. Simulation

In this part, we will evaluate our distributed algorithm for MOC-DVB by comparing it with other algorithms proposed for constructing VBs in terms of the size of VB, *Maximum Routing Cost* (MRC) representing the maximum routing cost between any pair of nodes, and *Average Routing Cost* (ARC) representing the average routing cost between any pair of nodes. In our simulations, the source node of every packet



(a) Transmission Range = 15 (b) Transmission Range = 20

Fig. 5. Comparison of number of directions selected in VBs using 90^o directional antennas, among CDS-BD-D, FKMS06, ZJH06, FlagContest, MOC-DVB, and ECC in UDG Networks.



(a) Transmission Range = 15 (b) Transmission Range = 20

Fig. 6. Comparison of number of directions selected in VBs using 22.5^o directional antennas, among CDS-BD-D, FKMS06, ZJH06, FlagContest, MOC-DVB, and ECC in UDG Networks.

can initiate the packet in any direction. Finally, the packet will be delivered to the destination through the direction selected in VBs. Our algorithm will be compared with FKMS06 [19], ZJH06 [20], CDS-BD-D [2], ECC [3], and ContestFlag [17].

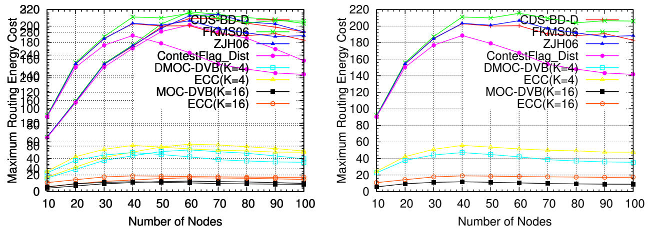
A. Simulation Environment

According to our network model introduced before, all nodes in a network share the same communication radius. However, in this part, we do not consider the existence of obstacles since other algorithms are proposed without consideration of obstacles.

n nodes are deployed randomly in a fixed area of $100m \times 100m$ and all nodes have the same transmission range. n is incremented from 10 to 100 by 10, while transmission ranges vary between $15m$ and $20m$. In addition, number of antennas K used by one node is 4 or 16, then the degree of the antennas is 90^o or 22.5^o respectively. For a certain, n, K, and transmission range, 100 instances are generated. Results are averaged among 100 instances.

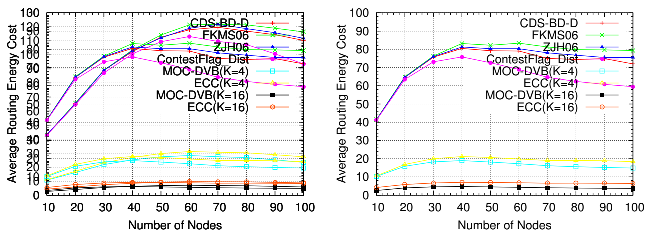
B. Simulation Results

Fig. 5 and Fig. 6 show the size of directions selected in VBs by using 4 uniform antennas on each node in Fig. 5, while using 16 uniform antennas in Fig. 6. On one hand, in FKMS06, ZJH064, CDS-BD-D, and ContestFlag, if one node is selected in the VB, then all directions of the nodes will be selected. The two figures show that our algorithm does not select too many directions compared to other VBs algorithms. Meanwhile, from Fig. 5 and Fig. 6, we can also tell that the difference between our MOC-DVB and other regular VB will increase greatly, when the degree of each antenna decreases. On the other hand, even though ECC selects a little bit fewer directions, the tradeoff is that ECC needs to compute all paths



(a) Transmission Range = 15 (b) Transmission Range = 20

Fig. 7. Comparison of Maximum Routing Cost among CDS-BD-D, FKMS06, ZJH06, ECC, and MOC-DVB in UDG Networks.



(a) Transmission Range = 15 (b) Transmission Range = 20

Fig. 8. Comparison of Average Routing Cost among CDS-BD-D, FKMS06, ZJH06, ECC, and MOC-DVB in UDG Networks.

with local information to select proper directions under some circumstances, which is not efficient.

As shown in Fig. 7 and Fig. 8, the MRC of FlagContest is about 50%-60% better and the ARC of FlagContest is around 65%-80% better. Note ARC and MRC increase first and then decrease. The reason is that in a connected network with small size of nodes, the routing path length is more likely to increase when a new node is added. For example, a network with 1 node inside has ARC equal to 0. When a new node is connected to the network, both ARC and MRC will increase to 1. Hence, routing path length increases when n increases (n is relatively small). However, when n exceeds a certain value, newly added nodes are more likely to make distance between nodes smaller and the network more connected (considering physical space is fixed) which explains both MRC and ARC decrease. In addition, when transmission range increases, networks are more connected considering physical space is fixed. This explains the decrease in Fig. 8 (b) and Fig. 7 (b) compared to that in Fig. 8 (a) and Fig. 7 (a) respectively.

VII. Conclusion

In this paper, we propose a minimum routing cost virtual backbone (MOC-DVB). MOC-DVB aims to find a minimum virtual (VB) backbone while assuring that any routing cost through this VB is smallest in networks. It is proved that selecting a minimum MOC-DVB is NP-hard in a general directed graph. An unreachable lower bound of approximation ratio of MOC-DVB is proved in this paper. We also propose an efficient distributed algorithm for constructing MOC-DVB with performance ratio $1 + \ln K + 2 \ln \delta_D$, where δ_D is the maximum *direction degree* in the network and K represents the number of antennas deployed on each node in the network. Compared with traditional VB, using a MOC-DVB as a virtual backbone in wireless networks can reduce routing cost significantly. Our future work includes further simulations in more realistic simulation environments like NS2.

Acknowledgment

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