Efficient Virtual Backbone Construction with Routing Cost Constraint in Wireless Networks using Directional Antennas

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Abstract—Directional antennas can divide the transmission range into several sectors. Thus, through switching off sectors in unnecessary directions in wireless networks, we can save bandwidth and energy consumption. In this paper, we will study a directional *virtual backbone* (VB) in the network where directional antennas are used. When constructing a VB, we will take routing and broadcasting into account since they are two common operations in wireless networks. Hence, we will study a VB with guaranteed routing costs, named α *M*inimum r*O*uting *C*ost *D*irectional VB (α-MOC-DVB). Besides the properties of regular VBs, α-MOC-DVB also has a special constraint — for any pair of nodes, there exists at least one path all intermediate directions on which must belong to α-MOC-DVB and the number of intermediate directions on the path is smaller than α times that on the shortest path. We prove that construction of a minimum α-MOC-DVB is an NP-hard problem in a general directed graph. A heuristic algorithm is proposed and theoretical analysis is also discussed in the paper. Extensive simulations demonstrate that our α-MOC-DVB is much more efficient in the sense of VB size and routing costs compared to other VBs.

Index Terms—Directional antennas; connected dominating set; routing costs; wireless network; obstacle; general graph; NP-hard; virtual backbone

1 INTRODUCTION

Wireless network has always been a hot topic in research community since it has a variety of military and civil applications such as environmental detections, health applications, disaster recoveries, etc. Due to its flexible deployment and mobile connectivity, it is believed that wireless network must act a vital part in the next generation network. However, different from the wired network, there are no underlying physical infrastructures in wireless networks. In order to enable data transfers in wireless network, all wireless nodes need to frequently *flood* control messages causing "broadcast storm problem" [1]. Thus, inspired by the physical backbone in wired networks, it is believed that a *V*irtual *B*ackbone (VB) [2] in the wireless network will help achieve efficient broadcasting.

In most virtual backbone research, *C*onnected *D*ominating *S*et (CDS) is selected to be a virtual backbone in wireless networks. If a network is modeled as $G = (V, E)$, where V represents the node set in the network and E represents the link set in the network, then CDS is a subset S of V satisfying the following two requirements: 1). Any node outside S has at least one edge incident on a node in S. 2). S can induce a connected subgraph in G.

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Due to the characteristics of CDS, forwardings only happen on nodes inside a CDS of a wireless network. On one hand, if the number of nodes inside a CDS is small, then the number of nodes involved in broadcasting will also be smaller. Correspondingly, redundancy and interference will be reduced. Hence, efficient CDS-based broadcasting is achieved. On the other hand, it is easier to maintain a CDS with smaller size. Routing path searching time and routing table size will be reduced too. Thus, most CDS researches focus on how to reduce the number of nodes selected to a CDS.

However, there are two drawbacks of prior CDS researches. The first one is the forwardings in unnecessary directions. If we divide the transmission range into several sectors, then some sectors of the selected nodes may have no receivers. As a result, a rather small portion of the transmission power is actually intercepted by the intended receivers. In this paper, we assume that every node in the network shares the same transmission radius. Transmission range is divided into non-overlapping uniform sectors. The forwarding energy cost of each antenna is the same because of the same radius and angle. Meanwhile, nodes can only receive the messages from the directions where they are in. The forwardings in those sectors having no receivers will waste energy and introduce more interferences to the network. In Fig. 1, the transmission range of each node is divided into four uniform sectors. If one node forwards a packet in two directions, then the energy cost will be twice as the case that the node forwards the

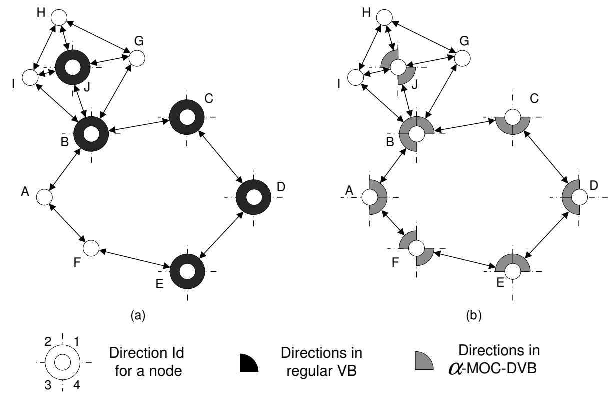


Fig. 1. Comparison between CDS and α-MOC-DVB

packet in only one direction. The ith direction of node u is denoted as $u(i)$. In Fig. 1 (a), B, C, D, E, and J construct a minimum CDS, where 20 sectors are selected. Actually, since there are no receivers in $B(4)$, $C(1)$, $C(2)$, $D(1)$, $D(4)$, $E(3)$, and $E(4)$, power spreading in these directions, where no intended receivers are, cannot make efficient use of power and even worse, collisions may happen in these redundant directions.

The second drawback is routing cost increase, that is, routing cost through CDS may increase a lot compared to the minimum routing cost in the network. Because CDS is a node subset S of a network, for any pair of nodes $u, v \notin S$, there may not exist a shortest path between u, v all of whose intermediate nodes belong to S. In CDS-based routing, a detour between u and v will be used and all intermediate nodes on this detour should belong to the CDS. Routing cost on one path can be calculated by the hop count on the path (a.k.a the length of the path) since every node has the same transmission range. More intermediate nodes on a path means more energy consumption on this path and lower packets' delivery ratio [3]. Also in Fig. 1 (a), there exists a bidirectional shortest path between A and E, denoted by $p(A-E) = \{A-F-E\}$ which means A's messages can arrive E by nodes on the path p except A and E. There are only 2 hops on $p(A-E)$. However, a detour through the minimum CDS should be $p^S(A-E) = \{A-B-C-D-E\}$ with four hops twice that on the shortest path $p(A-E)$.

From the above discussions, we can study how to achieve efficient VB-based routing from two aspects. The first one is to save energy in redundant directions. The second one is to consider routing cost constraint during VB construction process.

To avoid energy cost in unnecessary directions, [4] proposed a *Directional* CDS (DCDS) in a network using directional antennas. Instead of focusing on the node subset selection in regular CDS construction, DCDS focuses on selecting the sectors switched on forming a DCDS. Unnecessary directions will not be selected to DCDS. In a way, DCDS achieves energy efficiency by saving energy in unnecessary sectors. However, the routing path length is ignored in DCDS. Thus, in this paper, we will take both directional antennas and routing path length into CDS construction.

In [3], *diameter* is defined as the longest length among all shortest paths in the network. To better evaluate the quality of a CDS, *diameter* was used as an additional metric besides of CDS size. If the diameter of the induced subgraph of a CDS is small, then the maximum routing path through the CDS will be small too. Hence, such a CDS is regarded as an efficient CDS for routing. Later on, Kim *et al.* [5] proposed another concept *A*verage *B*ackbone *P*ath *L*ength (ABPL). ABPL denotes the average routing path length in a graph. In [5], ABPL is used to evaluate the efficiency of a CDS. Recently, [6] proposed the concept of MOC-CDS, where shortest path is considered. However, we can find that the constraint is so strict that the CDS size will be increased greatly.

Therefore, we propose another VB (α *M*inimum r*O*uting *C*ost *D*irectional VB α-MOC-DVB) to realize efficient broadcasting and routing, since the two operations are very common in wireless networks. The forwarding and reception model of directional antenna are directional forwarding and omni-reception, respectively. α-MOC-DVB is defined as a subset of directions in the graph, requiring that from any node to every other node in the graph, there exists at least one path all of whose intermediate directions belong to α-MOC-DVB and the number of intermediate directions on which should be smaller than α times that of the shortest path. In Fig. 1 (b), 15 directions — $A(1)$, $A(4)$, $B(1)$, $B(2)$, $B(3)$, $C(3)$, $C(4)$, $D(2)$, $D(3)$, $E(1)$, $E(2)$, $F(2)$, $F(4)$, $J(2)$, and $J(4)$, construct a minimum α-MOC-DVB, where $\alpha = 1$. From the example in Fig. 1 (b), not only the sectors where no receivers are (like C_1) but also some sectors having receivers (like $J(1)$) are switched off. Meanwhile, $B(1)$, $B(2)$, $B(3)$, $C(3)$, $C(4)$, $D(2)$, $D(3)$, $E(1)$, $E(2)$, $J(2)$, and $J(4)$ construct a minimum α-MOC-DVB, where $\alpha = 3$.

Our contributions in this paper are as follows:

1) A VB under routing cost constraint is proposed to achieve efficient VB-based routing and broadcasting, denoted as α-MOC-DVB. Different from prior VB construction, α-MOC-DVB focuses on selecting a direction subset with routing cost constraint.
2) We prove that construction of a minimum α-MOC-DVB is NP-hard in a general directed graph.
3) It is proved that the performance ratio of α-MOC-DVB has an unreachable lower bound of $\rho \ln \delta_D$, unless $NP \subseteq DTIME(n^{O(\log \log n)})$, where ρ is an arbitrary nonnegative number smaller than 1 and δ_D is the maximum *direction degree* in a graph.
4) We propose a distributed and approximation algorithm in this paper. We also prove that the size of the selected α-MOC-DVB is within $(1 + \ln K + 2 \ln \delta_D) * opt$, where *opt* is the size of the minimum α-MOC-DVB, K represents the number of uniform directions deployed on each node and $\alpha = 1$.

The rest of the paper will be organized as follows: in Section 2, we will recall the previous literatures on VB. The communication model and the formal definition of

α-MOC-DVB are given in Section 3. To simplify α-MOC-DVB, an equivalent problem (named α-2hop-DVB) to α-MOC-DVB will be introduced. It is proved that α-MOC-DVB is NP-hard. The corresponding heuristic algorithm is proposed in Section 4. The theoretical analysis is in Section 5. We will do thorough simulations in Section 6 to demonstrate the efficiency of α-MOC-DVB in broadcasting and routing. Finally, the paper will be concluded in Section 7.

2 Related Work

In wireless networks, there are several familiar models used in previous literatures — general graph [6], *U*nit *D*isk *G*raph (UDG), *D*isk *G*raph (DG) [7] and *Q*uasi *U*nit *D*isk *G*raph (QUDG). In [8], [9], they model the network as a QUDG, where obstacles [10] exist, no directional antennas are used and every node has the same transmission range. In this paper, we exploit directional antennas in the networks while keeping other two assumptions — obstacle existence and same transmission range. Hence, it is reasonable to model the network as a directed general graph in this paper.

*V*irtual *B*ackbone (VB) is a fundamental technique in wireless networks. Existing studies on VB construction mainly focus on *C*onnected *D*ominating *S*et (CDS) construction. Previous literatures on this topic focus on reducing the size of CDS. If the size of CDS is small, then the routing cost of CDS-based broadcasting will be small too because a few nodes in the network will be devoted to forwarding alleviating the the congestion in the network. From the applications of CDS, size cannot be the only issue we need to solve. However, when CDS size is too small, routing through CDS will detour through the nodes in CDS that excludes many intermediate nodes on shortest paths. As a result, routing costs will be increased.

There are two main categories of VB — one is VB without directional antennas and the other one is VB with directional antennas (DVB).

2.1 Virtual Backbone

As a fundamental issue in wireless networks, VB can always attract attention from the research community. Most existing works on VB are CDS constructions. In [11], a minimum CDS is proved NP-hard in a general graph. Later on in [12], Lichtenstein proved that a minimum CDS is NP-hard even in a UDG.

There is a way to classify previous literatures on CDS based on the construction process. The first one is 2-stage and the second one is 1-stage.

We can further classify 2-stage literatures into two subtypes [13]. The first one is pruning based CDS construction. The other one is a *D*ominating *S*et (DS) based CDS construction. In the first subtype, a CDS is constructed firstly with more redundant nodes in the first stage. The task of the second stage is to remove the redundant nodes selected in the previous stage as many as possible. The typical algorithm, belonging to this subtype, is in [14]. They proved that the approximation ratio of their algorithm is $O(n)$, where n is the number of nodes in the network. Contrarily, in the second subtype, a DS is constructed firstly and more nodes will be added to make a CDS. In [15], the authors proposed an algorithm belonging to the second subtype and the performance ratio of the algorithm is $H(\delta) + 2$ where H is harmonic function and δ is the maximum node degree in the network. A leader algorithm of the second subtype is proposed by Butenko *et al.* [16]. The size of the selected CDS is smaller than $8|opt| + 1$, where opt is the size of a minimum CDS. Actually, a *M*aximum *I*ndependent *S*et (MIS) [17] in a graph is also a DS. Hence, MIS and Steiner Tree [18] are used in [19] achieving a CDS with size smaller than $3.8|opt| + 1.2$.

Different from the 2-stage algorithm, 1-stage algorithm is to find a CDS directly without the stage of DS or redundant CDS. Guha *et al.* also proposed a 1-stage algorithm yielding approximation ratio of $2H(\delta)+2$. Later, Ruan *et al*. [20] made a modification of the selection standard of DS in [15]. Thus, the 2-stage algorithm in [15] is reduced to a 1-stage algorithm with approximation ratio of $3 + \ln \delta$.

2.2 Directional Virtual Backbone

As we all know, the application of directional antennas can save energy and reduce collisions in the network. Hence, some VB researches are done with directional antennas to achieve efficient VB-based routing and broadcasting. In [4], Yang *et al.* propose the concept of *D*irectional CDS (DCDS). In Directional Virtual Backbone (DVB), the goal is to find the directions as few as possible to construct a VB. DCDS is proved NP-hard in [4]. Correspondingly, a localized heuristic algorithm is proposed in [4]. However, the time complexity in this paper is exponential under some circumstances since they need to compute all paths between any two nodes to make a decision whether one direction is selected or not in the worst case. [6], [21] study VB with guaranteed routing costs. However, CDS size is too large. Hence, routing costs will be increased too.

Besides routing [22] and broadcasting [23], virtual backbone has many other applications (e.g., topology control [24]) in wireless networks. In this paper, we mainly focus on how to construct a VB yielding both efficient routing and efficient broadcasting.

3 Problem Statement

In this section, we will introduce the directional antennas used in this paper. The network model will be introduced, where directional antennas are exploited. Then α-MOC-DVB will be introduced formally. To simplify the problem of α-MOC-DVB, an equivalent problem α-2hop-DVB will be studied. We will prove that α-2hop-DVB is NP-hard in the network model used in this paper.

3.1 Directional Antenna

The techniques used in smart antenna system are illustrated in [25]. In the antenna system, directional transmission and reception are possible. We will introduce the forwarding and reception strategies in the following part, respectively.

Assume that the directional antennas are regular, aligned and nonoverlapped. The messages can be sent out through the selected antennas with the switched beam technique. In this paper, we assume that in one network, uniform directional antennas are used. Then the transmission range of each node is divided into several uniform sectors. In this paper, we denote the ith sector of node u as $u(i)$. One node can only send out messages through its sectors switched on. And one node can only receive messages from the sectors where it is. In Fig. 2 (a), A's transmission range is divided into 4 sectors. And node C is in $A(4)$. Hence, C can only receive messages sent out in $A(4)$, instead of any other sectors of A. If $A(4)$ is switched on, then A can send message to C. Otherwise, A cannot reach C directly without help of other forwarders.

In directional antenna systems, two reception techniques are used. One is directional reception. In this technique, some directions are predetermined used for reception. Nodes cannot receive messages from those directions not predetermined. The other one is omni-reception. In this technique, nodes can receive messages from its neighbors in any direction. For convenience, we will use the second reception technique in this paper.

3.2 Network Model

In this paper, we consider the existence of obstacles [10]. In wireless networks, communications are realized by the radio wave transmissions. Obstacles can stop the communications between any two nodes by four kinds of influence — scattering, reflection, diffraction, and blocking [10]. Every two nodes u and v can communicate with each other when and only when three requirements are satisfied at the same time — (a). u and v are in each other's transmission range. (b). At least one sector of u where v is is switched on and at least one sector of v where u is is switched on. (c). Neither of transmissions from u to v or from v to u is forbidden by the obstacles.

We also assume that the transmission radius for each node is the same. For any two nodes u and v, if there is a directional link from u to v and v is in $u(i)$, then we will use a directed edge $u(i) \to v$ to denote it. $u(i) \to v$ means that u can forward the messages to v through its ith direction when u receive messages from other nodes by the technique of omni-reception. Taking all the situations we assume above into consideration, it is quite reasonable to model a network as a directed graph $G = (V, E, D)$, where V is the node set in the network, E is the directed edge set in the network, and D is the sector set in the network. In this paper, we assume that G is a strongly connected directed graph. A directed graph is called strongly connected if there is a path from each node in the graph to every other node.

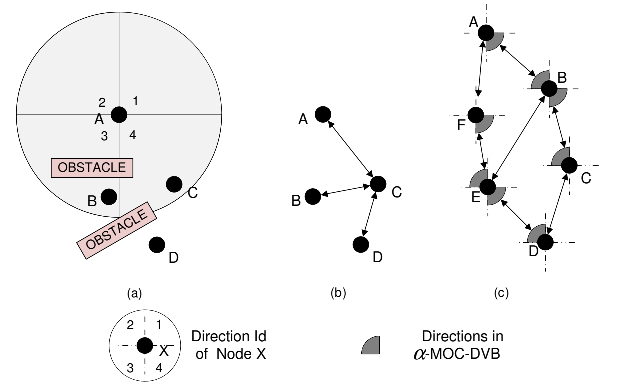


Fig. 2. Network Model and Illustration of α-MOC-DVB. (a). The network has obstacles. (b). The directed graph of the network. (c). Illustration of α-MOC-DVB property.

In Fig. 2 (a), B is in the sector $A(3)$ and then A must be in at least one of B's sectors since all sectors have same radius. However, because of the existence of obstacle between A and B, the link between A and B is cut off. Same thing happens on the link between B and D. Thus, we have the induced graph in Fig. 2 (b).

3.3 Energy Consumption Model

We assume that the transmission radius in one network is r. The energy consumption (denoted as EC_s) of one forwarding in one sector s is proportional to r and the angle (θ) of the sector. We have $EC_s = C * r * \theta$, where C is a constant value [23]. The energy cost of a path is the sum of cost of all forwardings on the path. Since r and θ are fixed in one network, the forwarding in one sector in the network will be fixed. Thus, the energy cost of one path is determined by hops on the path in one network.

3.4 Problem Definition

Since G is a strongly connected graph, then from any node x to every other node y, there must be at least one directed path $p(x \to y) = \{x \to w_1(w_2) \to w_2(w_3)... \to w_k(y) \to y\}$, where $w_i(w_{i+1})$ represents the sector of w_i which w_i uses to forward messages to w_{i+1}. The shortest path from x to y is defined as the path having the smallest number of intermediate directions among all paths. $H(x \to y)$ denotes the hop count on the shortest path from x to y. In wireless network, longer routing path will decrease the delivery ratio and increase interference. To achieve efficient routing, we study α-MOC-DVB in this paper. The formal definition of α-MOC-DVB is given in Def. 1.

Definition 1 (α-MOC-DVB)**.** *The* α Minimum rOuting Cost Directional Virtual Backbone *problem (α-MOC-DVB) is to find a direction set $D_{Sub} \subseteq D$ in $G = (V, E, D)$ such that*

1) D_{Sub} *can induce a node subset* V_{Sub} *where every node in* V_{Sub} *has at least one sector in* D_{Sub}*.* $\forall x \notin V_{Sub}$*, there exists a* y *in* V_{Sub} *and the sector of* y *where* x *is is in* D_{Sub}*.*
2) $\forall\ x, y \in V$*, if* $H(x \to y) > 1$*,* $\exists\ p(x \to y)$*, all intermediate directions on* $p(x \to y)$ *belong to* D_{Sub} *and the number of intermediate directions on* $p(x \to y)$ *should be smaller than* α *times that on the shortest path from* x *to* y*.*

Actually, we do not require that D_{Sub} induces a connected subgraph. However, we can still deliver messages successfully through directions in D_{Sub} if the messages are initiated by the source node in omni-directions. In Fig. 2 (c), only through the directions in VB, A cannot reach D. Hence, the selected directions cannot induce a connected subgraph. However, messages can still be delivered successfully if A initiates messages in omni-directions. $p(A \to D) = \{A \to F(4) \to E(4) \to D\}$ while $p(D \to A) = \{D \to C(2) \to B(2) \to A\}$, where both A and D initiate messages in omni-directions.

In α-MOC-DVB, we do not need to consider the situation of the pair of nodes which can communicate with each other directly in G without other nodes' helps. For the special case of a complete graph, one node will be selected arbitrarily, and switch on its sectors having receivers.

To simplify the construction of an α-MOC-DVB, we find an equivalent VB to α-MOC-DVB, named α-2hop-DVB formally defined in Def. 2. We prove the equivalence of the two types of VB in Lemma 1.

Definition 2 (α-2hop-DVB)**.** *The* α 2hop Directional Virtual Backbone *problem (*α*-2hop-DVB) is to find a direction set* $D'_{Sub} \subseteq D$ *in* $G = (V, E, D)$ *such that*

1) D'_{Sub} *can induce a node subset* V'_{Sub} *where every node in* V'_{Sub} *has at least one sector in* D'_{Sub}*.* $\forall x \notin V'_{Sub}$*, there exists a* y *in* V'_{Sub} *and the sector of* y *where* x *is is in* D'_{Sub}*.*
2) $\forall\ x, y \in V$*, if* $H(x \to y) = 2$*,* $\exists\ p(x \to y)$*, all intermediate directions on* $p(x \to y)$ *belong to* D'_{Sub} *and the number of intermediate directions on* $p(x \to y)$ *should be smaller than* α*.*

Lemma 1. *α-MOC-DVB and α-2hop-DVB are equivalent to each other.*

Proof: We will prove this lemma from two directions — an α-MOC-DVB is an α-2hop-DVB ("$\Rightarrow$") and an α-2hop-DVB is also an α-MOC-DVB ("$\Leftarrow$").

("$\Rightarrow$") If a direction set $DirS$ is an α-MOC-DVB, then it must satisfy the dominating attribute. Thus, $DirS$ also meets the first constraint in α-2hop-DVB. Moreover, $DirS$ also guarantees the number of intermediate directions on the routing path from any node to another node with distance bigger than 1. Hence, $DirS$ must guarantees the intermediator number on the path from any node to another node with two hops away. This satisfies the second constraint in α-2hop-DVB. Thus, $DirS$ is also an α-2hop-DVB.

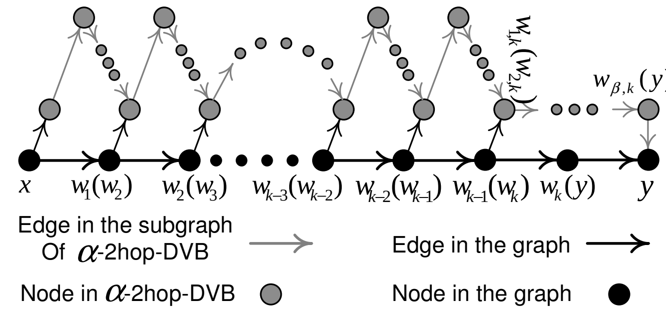


Fig. 3. Equivalence between MOC-DVB and 2hop-DVB

("$\Leftarrow$") If a direction set $DirS$ is an α-2hop-DVB, then it must satisfy the first constraint (dominating attribute) in α-MOC-DVB. We also need to prove that $DirS$ also satisfies the second constraint in α-MOC-DVB.

We select any two nodes (x and y in Fig. 3) arbitrarily from the graph having $H(x \to y) = k > 1$, there must exist a shortest path from x to y — $p(x \to y) = \{x \to w_1(w_2) \to w_2(w_3) \to \ldots \to w_k(y) \to y\}$. We can tell that $H(w_{k-1} \to y) = 2$. Hence, we can find at most α directions from $DirS$ to replace the direction $w_k(y)$ — $w_{1,k}(w_{2,k})$, ..., $w_{\beta,k}(y)$ of nodes $w_{1,k}$,..., $w_{\beta,k}$ respectively ($\beta \leq \alpha$). We can get a replacement path $p^k(x \to y) = \{x \to w_1(x) \to w_2(w_1) \to \ldots \to w_{k-1}(w_{1,k}) \to w_{1,k}(w_{2,k}), \ldots, \to w_{\beta,k}(y) \to y\}$. Continue, we can tell $H(w_{k-2} \to w_{1,k}) = 2$ from $p^k(x \to y)$. At most α directions can be found in $DirS$ to replace w_{k-1}. Hence, for each intermediate direction in $p(x \to y)$, we can find at most α replacement directions in $DirS$. Finally, we can get a replacement path where all intermediate directions are in $DirS$ and the number of directions on the replacement path is smaller than or equal to $\alpha * k$. Hence, $DirS$ also satisfies the second constraint in α-MOC-DVB.

In sum, one direction subset $DirS$ is an α-MOC-DVB if and only if it is an α-2hop-DVB. □

From the proof of equivalence (part "$\Leftarrow$"), we know that for any path in the network, we can find a replacement path with all directions in α-2hop-DVB except the initiation directions. Thus, we can conclude that if we assume that source nodes initiate messages in all directions then we can get successful routing and broadcasting only through directions in the DVB.

α-2hop-DVB-360^o is a special case where omnidirectional antennas are used. α-2hop-DVB is a generalization of α-2hop-DVB-360^o. Thus, if α-2hop-DVB-360^o is NP-hard, then the generalization α-2hop-DVB must be NP-hard. Before we prove that the α-2hop-DVB-360^o is NP-hard, we first introduce α-2hop-DS which has been prove NP-hard in [21].

Definition 3 (α-2hop-DS [21])**.** *Given a strongly connected bi-directed graph* $G_{bi} = (V_{bi}, E_{bi})$ *where* V_{bi} *is the node set and* E_{bi} *is the edge set, the* α*-*2hop Minimum rOuting Cost Dominating Set *(*α*-2hop-DS) is a node set* $S_{bi} \subseteq V_{bi}$ *such that*

1) $\forall\ u_{bi} \in V_{bi} \backslash D_{bi}$, $\exists\ v_{bi} \in D_{bi}$, *such that* $(u_{bi}, v_{bi}) \in E_{bi}$.
2) $\forall\ u_{bi}, v_{bi} \in V_{bi}$, *if* $Dist(u_{bi}, v_{bi}) = 2$, *then* $\exists\ p^{D_{bi}}(u_{bi}, v_{bi})$ *on which all intermediate nodes belong to* D_{bi} *and* $m_D(u_{bi}, v_{bi}) \leq \alpha * m(u_{bi}, v_{bi})$, *where* $m_D(u_{bi}, v_{bi})$ *and* $m(u_{bi}, v_{bi}) = 1$ *are the number of intermediate nodes on* $p^D(u_{bi}, v_{bi})$ *and* $p_{shortest}(u_{bi}, v_{bi})$ *respectively.*

Lemma 2. *α-2hop-DVB-360^o is NP-hard in a directed graph, $\forall \alpha \geq 1$.*

Proof: It suffices to show the following decision version of α-2hop-DVB-360^o is NP-hard.

> DECISION VERSION OF α-2HOP-DVB-360^o: Given a directed graph G and a positive integer k, determine whether G has an α-2hop-DVB-360^o of size at most k.

To do so, we reduce the following decision version of α-2hop-DS problem to DECISION VERSION OF α-2HOP-DVB-360^o.

> DECISION VERSION OF α-2HOP-DS: Given a bi-directed graph $G_{bi} = (V_{bi}, E_{bi})$ and a positive integer h, determine whether G_{bi} has an α-2hop-DS of size at most h.

$\forall v_{bi} \in G_{bi}$, one 360^o directional antenna is deployed. And derive two directed edges from one bi-directed edge. Then, we can derive a directed graph $G = (V, E, D)$. $V = V_{bi}$. $|E| = 2|E_{bi}|$ and $\forall (x, y) \in E_{bi}$, $\exists (x \rightarrow y) \in E$ and $(y \rightarrow x) \in E$. $\forall x \in V_{bi}$, $\exists d_x \in D$ and the degree of d_x is 360^o.

We will show that G has an α-2hop-DVB-360^o of size at most k if and only if G_{bi} has an α-2hop-DS of size at most $h = k$.

("$\Rightarrow$") We first prove that when the size of the α-2hop-DVB-360^o (D_{Sub}) in G is at most k, then we can derive an α-2hop-DS (S_{bi}) in G_{bi} of size at most k. In the definition of α-2hop-DVB-360^o, we can get a node subset V_{Sub} from D_{Sub} and the cardinality of V_{Sub} is at most k. We will prove that $V_{Sub} \subseteq V_{bi}$ is an α-2hop-DS. Any node x outside V_{Sub} will have an adjacent node y having the direction $y(x) \in D_{Sub}$ and $y(x) \rightarrow x \in E$. In fact, if $(s \rightarrow t) \in E$, then $(s \leftrightarrow t) \in E_{bi}$. Thus, $\forall x \notin V_{Sub}$, $\exists y \in V_{Sub}$ having $(x \leftrightarrow y) \in E_{bi}$. From any node x to another node y having $h(x \rightarrow y) = 2 \in G$, there exists one directional path $p(x \rightarrow y) = \{x \rightarrow w_1(w_2) \rightarrow ... \rightarrow w_\beta(y) \rightarrow y\}$, where $\beta \leq \alpha$, satisfying "α" constraint and all intermediate nodes on which belong to D_{Sub}. Moreover, we have $w_1, ..., w_\beta \in V_{Sub}$. Actually, if $H(x \rightarrow y) = 2 \in G$ then $H(x \leftrightarrow y) = 2 \in G_{bi}$. Meanwhile, we have one bidirectional path $p_{bi}(x \leftrightarrow y) = \{x \leftrightarrow w_1 \leftrightarrow ... \leftrightarrow w_\beta \leftrightarrow y\} \in G_{bi}$, satisfying α constraint. Therefore, we prove that V_{Sub} is an α-2hop-DS of size at most k.

("$\Leftarrow$") In this part, we will prove when the size of the α-2hop-DS (S_{bi}) is at most k, then we can derive an α-2hop-DVB-360^o in G of size at most k. For every node in S_{bi}, its sole direction (360^o) will be included in D_{Sub} and no more other directions are in D_{Sub}. Hence the size of D_{Sub} is at most k. We need to further prove D_{Sub} satisfies the definition of α-2hop-DVB-360^o. $\forall x \notin S_{bi}$, $\exists y \in S_{bi}$ having $x \leftrightarrow y \in E_{bi}$. Correspondingly, we must have $y \rightarrow x \in E$ and $y(x) \in D_{Sub}$ because the direction on each node is 360^o. "Dominating" is proved. For any pair x and y having $H(x \leftrightarrow y) = 2 \in G_{bi}$, we have one bidirectional path $p_{bi} = \{x \leftrightarrow w_1 \leftrightarrow ... \leftrightarrow w_\beta \leftrightarrow y\} \in G_{bi}$, where $\beta \leq \alpha$. Since every node selected in α-2hop-DS has one 360^o direction in D_{Sub}, we can derive two directional paths in G — $p(x \rightarrow y) = \{x \rightarrow w_1(w_2) \rightarrow ... \rightarrow w_\beta(y) \rightarrow y\}$ and $p(y \rightarrow x) = \{y \rightarrow w_\beta(w_{\beta-1}) \rightarrow ... \rightarrow w_1(x) \rightarrow x\}$ satisfying "α" constraint and all intermediate directions are in D_{Sub}, where $w_i(w_{i+1}) = w_i(w_{i-1})$. Therefore, D_{Sub} is an α-2hop-DVB-360^o.

In sum, α-2hop-DVB-360^o is NP-hard. □

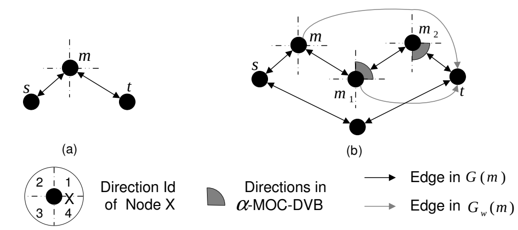


Fig. 4. Illustration of G and G_w in Alg. 1

α-2hop-DVB is generalization of α-2hop-DVB-360^o. Thus, α-2hop-DVB is NP-hard because of Lemma 2. Thus, we can get Theorem 1.

Theorem 1. *The α-MOC-DVB is NP-hard, $\forall \alpha \geq 1$.*

4 ALGORITHM

Inspired by α-2hop-DVB, we will only consider the pair of nodes exactly 2-hop away in our algorithm. The algorithm that we propose is heuristic and localized algorithm. Since the uniform directional antennas are used, we try to select the directions acts as the intermediate directions as more time as possible.

In our algorithm, we assign every node m a unique id, denoted as $ID(m)$. We also assign a unique id to each direction d, denoted as $ID(d)$.

Every decision on each direction d of each node m is based on its $\alpha + 1$ local topology information, denoted by $G(m) = (V(m), E(m), D(m))$. $G(m)$ is a directed and unweighted graph. Based on $G(m)$, we can derive a local weighted graph $G_w(m) = (V_w(m), E_w(m), D_w(m))$, where $V_w(m) = V(m)$ and $D_w(m) = D(m)$. If $e(v(v \rightarrow u) \rightarrow u) \in E(m)$, we have $e_w(v(v \rightarrow u) \rightarrow u) = 0 \in E_w(m)$, where $v(v \rightarrow u)$ represents the direction v uses to reach u.

Based on $G(m)$ and $G_w(m)$, we can calculate a pair set for each direction d of m, denoted as $pair(d)$. $pair(d)$ will record the pairs of nodes $s \in V_w(m)$, $t \in V_w(m)$ satisfying $H(s \rightarrow t) = 2 \in G(m)$ which means we also have $H(s \rightarrow t) = 2 \in G_w(m)$, $p_w(s \rightarrow t) = \{s(s \rightarrow m) \rightarrow m(m \rightarrow t) \rightarrow t\} \in G_w(m)$, $s(s \rightarrow m) \rightarrow m \in G_w(m)$, $m(m \rightarrow t) \rightarrow t \in G_w(m)$, and

$e_w(s(s \to m) \to m) + e_w(m(m \to t) \to t) + 1 \leq \alpha$, where $m(m \to t) = d$. $m(m \to t)$ does not mean that t must be in the direction d of node m. Actually, it means that m can reach t through directions already in VB by sending out messages in direction d. There are two situations that the d can reach t as shown in Fig. 4, where $d = m(4)$. In Fig. 4 (a), m can reach t through d directly. In Fig. 4 (b), m uses d to send messages to m_1 and then m_1 can reach t through directions already in VB. The two gray edges in Fig. 4 (b) are not there initially. They are added during **Local Weighted Subgraph's Update**. $f(d)$ denotes the cardinality of $pair(d)$, having $f(d) = |pair(d)|$. The definition of direction weight will be illustrated based on two elements — pair set and ID of the direction.

Algorithm 1 Distributed Selection of 2hop-DVB

Step 1. Each direction d with nonempty $pair(d)$, calculates $f(d) = |pair(d)|$. Send out $pair(d)$, $f(d)$, and $ID(d)$.

Step 2. Each node x will order directions whose information is sent out in Step 1 based on **Direction Weight** decreasingly. It sends flags following the policy of **Flag Sending Condition**. If there are more than one such d, then it breaks tie by choosing the one with lowest node id or lowest direction id;

Step 3. If a direction d receives flags from all its neighbors, it adds d to D_{Sub} and adds corresponding node to V_{Sub}. Update the local subgraph based on **Local Weighted Subgraph's Update**. Then it sends $pair(d)$ and the update information to all its neighbors within $\alpha+1$ hops. Set $pair(d) = \phi$;

Step 4. If node y receives $pair(d)$ and update information, update $G_w(y)$ firstly. There are three types of pairs which should be updated for each direction's $pair$ set in y — two kinds of "remove" actions and one kind of "add" action. 1). Add those pairs m_1, m_2 into $pair(dy)$ satisfying $H(m_1 \to m_2) = 2 \in G_w(y)$ and $e_w(m_1(m_1 \to y) \to y) + e_w(dy \to m_2) + 1 \leq \alpha$, where dy is any direction of node y. 2). Meanwhile, remove those pairs of nodes $(r_1, r_2) \in pair(dy)$, if $e_w(r_1(r_1 \to x) \to x) + e_w(x(x \to r_2) \to r_2) + 1 \leq \alpha$ and direction $x(x \to r_2)$ is already in the DVB. 3). y also computes union U of the received $pair(d)'s$ from other directions and updates $pair$ set of all directions of y by removing all pairs in U.

Direction Weight: *Given two direction d_1 and d_2, $W(d_1)$ and $W(d_2)$ denote the weight of d_1 and d_2, respectively. We say $W(d_1) < W(d_2)$ if and only if 1). $f(d_1) < f(d_2)$ or 2). $f(d_1) = f(d_2)$ and $ID(d_1) < ID(d_2)$.*

When one node collect pair sets and corresponding IDs from its neighbors, it will order all the directions, including its own pair sets, in an decreasing way based on the direction's weight. And send out flags following **Flag Sending Condition**. In our algorithm, one node may send out more than one flag at one time.

Flag Sending Condition: *A flag will be sent to direction dy from node m if and only if $pair(dx) \bigcap pair(dy) = \phi$, $\forall dx$ 1-hop away from m, having $W_{dx} > W_{dy}$.*

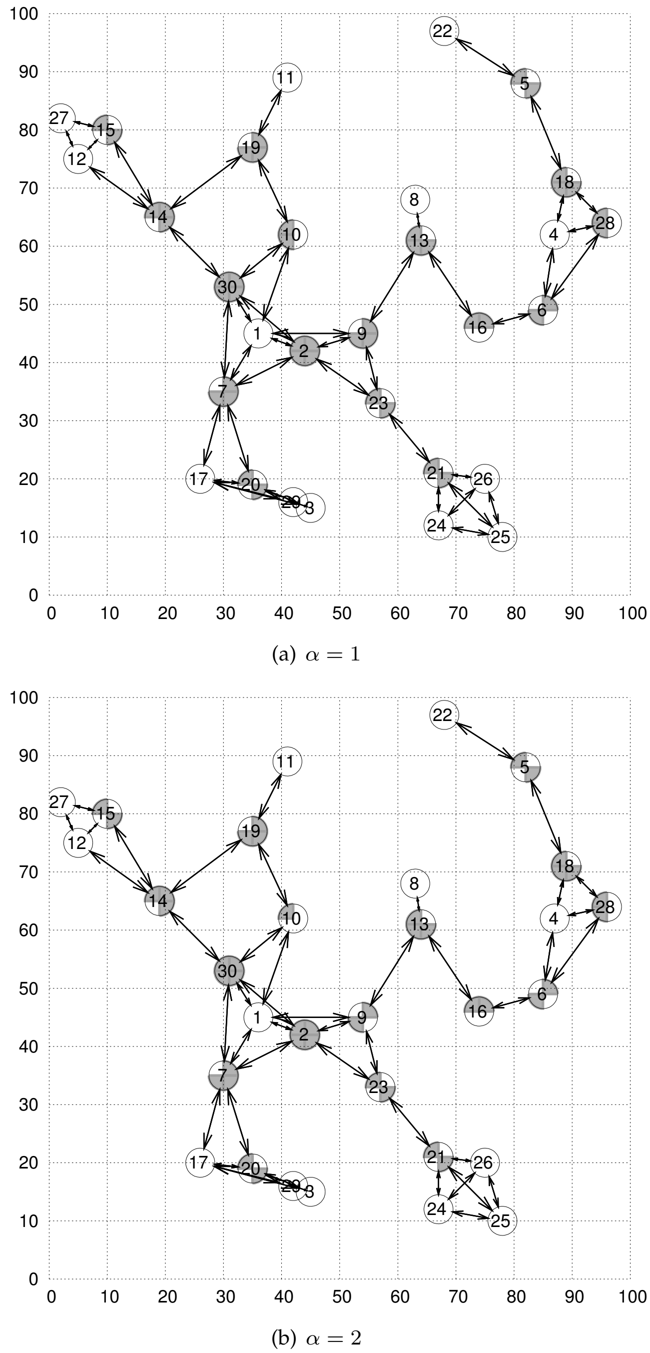
After one direction of one node receives all flags from its 1-hop neighbor nodes, the direction will be selected and colored black. And the local weighted subgraph of the node will be updated. In addition, the update information will be sent out $\alpha + 1$ hops away from the node.

Local Weighted Subgraph's Update: *If one direction dm of the node m is selected as a VB member, $G_w(m)$ will be updated. Add or update the edge $e_w(s(s \to t) \to t) = e_w(s(s \to m) \to m) + e_w(m(m \to t) \to t) + 1$ to $G_w(m)$ if the following four conditions are satisfied — 1). For any pair of nodes s and t having $H(s \to t) \leq 2 \in G_w(m)$. 2). $e_w(s(s \to m) \to m) + e_w(m(m \to t) \to t) + 1 < \alpha$. 3). $e_w(s(s \to m) \to m) + e_w(m(m \to t) \to t) + 1 < e_w(s(s \to t) \to t)$, if $e_w(s(s \to t) \to t)$ has already been in $G_w(m)$. 4). $m(m \to t) = dm$.*

The new edges added in **Local Weighted Subgraph's Update** will be passed away $(\alpha + 1)$-hop away. Nodes receive the update information will add the new edges to their own local weighted subgraphs.

The basic idea of the algorithm introduced in this paper is a greedy strategy. For each direction d, it will store $pair(d)$. At each step, we choose the direction which has the maximum cardinality of pair set. When one direction is selected, remove those pairs of nodes covered by the selected directions from those directions outside DVB. Continue selection until $pair(d) = \phi$ for all directions of all nodes. The details are given in Alg. 1.

Fig. 5 simulate a network in a region of $100m \times 100m$. 30 nodes are placed in the region. Transmission range of every node in the virtual network is $25m$ and four uniform directional antennas are deployed on each node of 90^o. Grey sectors, as shown in the figure, represent an α-MOC-DVB selected by Alg. 1. $\alpha = 1$ in Fig. 5 (a) while $\alpha = 2$ in Fig. 5 (b). 44 directions are selected in the resultant graph of 1-MOC-DVB and 42 one are selected in 2-MOC-DVB. For example, $f(N_{12}(2)) = f(N_{15}(2))$ and $ID(N_{12}(2)) < ID(N_{15}(2))$. Thus, $W(N_{12}(2)) < W(N_{15}(2))$. The flags will be sent to $N_{15}(2)$ instead of to $N_{12}(2)$. Hence, the second direction of 15 is selected firstly to 1-MOC-DVB. Then the set $pair$ of direction $N_{12}(2)$ will be recalculated as ϕ. As a result, the second direction of 12 will not be selected. $N_{10}(3)$ is selected in Fig. 5 (a) instead of in Fig. 5 (b) because of the relaxation on α. The pairs (N_{19}, N_1) and (N_{19}, N_{30}) are the only two pairs going through $N_{10}(3)$ having $H(N_{19} \to N_1) = 2$ and $H(N_{19} \to N_{30}) = 2$. Due to the relaxation, we can use the detour $p(N_{19} \to N_1) = \{N_{19} \to N_{14}(4) \to N_{30}(4) \to N_1\}$. And N_{19} can reach N_{30} through direction $N_{14}(4)$. Since $N_{14}(4)$ can also cover other pairs within 2-hops away, $N_{14}(4)$ is selected before $N_{10}(3)$. Thus, $N_{10}(3)$ will not be selected.



(a) $\alpha = 1$

(b) $\alpha = 2$

Fig. 5. An example of MOC-DVB by Alg. 1

5 THEORETICAL ANALYSIS

We will prove that our heuristic algorithm does output an α-MOC-DVB in Theorem 2. The performance ratio of our algorithm will also be studied. We will prove that there is an unreachable lower bound of α-MOC-DVB.

Theorem 2. *The direction subset selected D_{Sub} in Alg. 1 is an α-MOC-DVB.*

Proof: Since the original graph is strongly connected, then there must exist a node y 2-hop away for any node x. Initially, there should be a directional path $p(y \to x) = \{y \to m(x) \to x\}$ in $G(m)$ and $(y, x) \in pair(m(x))$. The pair (y, x) will not be removed from $pair(m(x))$ until there is a directional path $p(y \to x) = \{y \to m_1(m_2) \to \ldots \to m_\beta(x) \to x\}$ where $\beta \leq \alpha$ and all intermediate directions are in D_{Sub}. Hence w_k has a direction in D_{Sub} and x is in the direction. As a result, every node is dominated. And the number of intermediate directions will not exceed α from one node to another node within 2-hop away.

In sum, the selected directions by Alg. 1 form an α-2hop-DVB. Since α-2hop-DVB and α-MOC-DVB are equivalent to each other, the direction subset is also an α-MOC-DVB.

□

Theorem 3. *The message complexity of Alg. 1 is $O(\delta^{\alpha+1} * |V| * K + |V|^2 * K * \delta)$ under unicast transmission model, where δ is the maximum node degree in the graph and K denotes the number of uniform antennas deployed on each node.*

Proof: In Step 1 of Alg. 1, the message complexity at each round will be $O(\delta * |V|)$. The total message complexity of the first step is $O(rounds * |V| * \delta)$, where $rounds$ denotes the number of rounds that Alg. 1 will be done. In Step 2 of Alg. 1, each node will only send out at most one flag at each round. The total message complexity of the second step is $O(rounds * |V|)$. In Step 3 of Alg. 1, one update information will be propagated $\alpha + 1$ away. Hence, the message complexity in Step 3 is $O(\delta^{\alpha+1} * |V| * K)$. In the last step of the algorithm, no message will be sent out. We also have $rounds \leq K * |V|$.

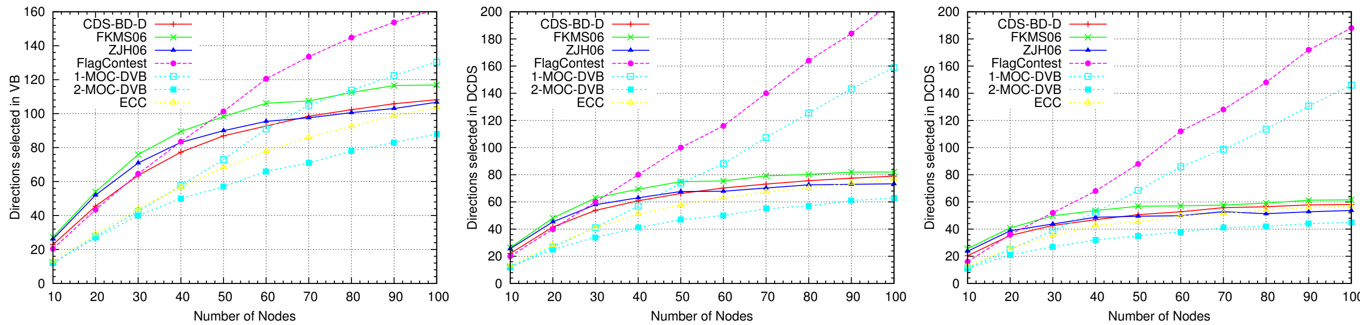
As a result, the total message complexity is $O(\delta^{\alpha+1} * |V| * K + |V|^2 * K * \delta)$. □

In [21], Ding *et al.* proved that there exists an unreachable lower bound of 1-2hop-DS — $\rho \ln \delta$, where $\forall \rho < 1$ and δ is the maximum node degree in a graph, unless $NP \subseteq DTIME(n^{O(\log \log n)})$. Thus, the generalization case α-2hop-DS ($\alpha \geq 1$) also has such an unreachable lower bound. We will prove that there also exists an unreachable lower bound of α-MOC-DVB.

Theorem 4. *Neither α-MOC-DVB nor α-2hop-DVB has a polynomial time algorithm with approximation ratio $\rho \ln \delta_D$, where $\forall \rho < 1$ and δ_D is the maximum* direction degree *in the input graph, unless $NP \subseteq DTIME(n^{O(\log \log n)})$.*

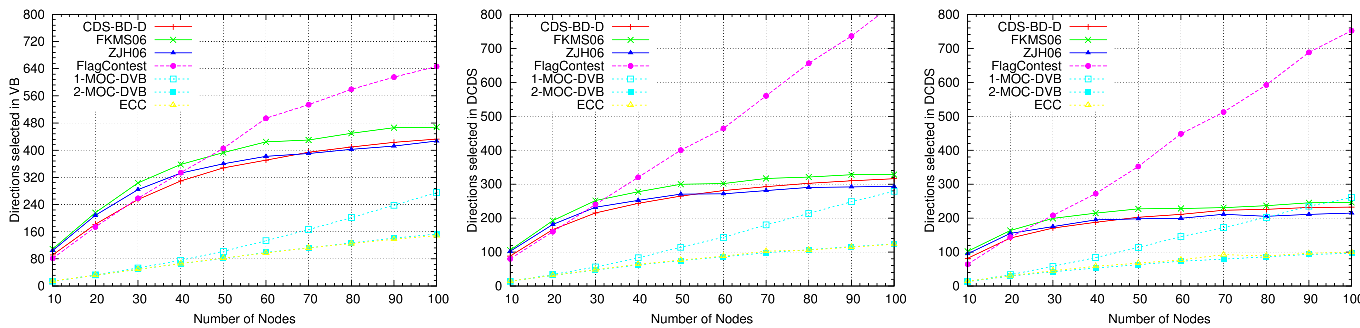
Proof: Based on the proof of Lemma 2, an immediate corollary of our claim is that the size of the minimum α-2hop-DS in the graph G_{bi} is $opt_{\alpha-2hop-DS}$ if and only if the size of the minimum 2hop-DVB-360^o in the corresponding graph G is $opt_{\alpha-2hop-DVB-360^o}$, where $opt_{\alpha-2hop-DS} = opt_{\alpha-2hop-DVB-360^o}$. We prove that we cannot propose a polynomial time algorithm to construct a 2hop-DVB-360^o with approximation ratio of $\rho \ln \delta_D$ by contradiction method.

Assume G has a polynomial time solution D_{Sub} to α-2hop-DVB-360^o with size at most $(\rho \ln \delta_D)(opt_{\alpha-2hop-DVB-360^o})$ for some constant $\rho < 1$. Thus, we can find a polynomial time solution to α-2hop-CDS with size at most $(\rho \ln \delta)(opt_{\alpha-2hop-DS})$. This implies that $NP \subseteq DTIME(n^{O(\log \log n)})$. Therefore, the



(a) Transmission Range = 20 (b) Transmission Range = 25 (c) Transmission Range = 30

Fig. 6. Comparison of VB size among CDS-BD-D, MFKMS06, ZJH06, FlagContest and α-MOC-DVB in UDG Networks, when $k = 4$.



(a) Transmission Range = 20 (b) Transmission Range = 25 (c) Transmission Range = 30

Fig. 7. Comparison of VB size among CDS-BD-D, MFKMS06, ZJH06, FlagContest and α-MOC-DVB in UDG Networks, when $k = 16$.

assumption that G has a polynomial time solution with size at most $(\rho \ln \delta_D)(opt_{\alpha-2hop-DVB-360^o})$ for some constant $\rho < 1$ is incorrect.

Since α-2hop-DVB-360^o is a special case of α-2hop-DVB, we can get that there dose not exist a polynomial time solution to α-2hop-DVB with size at most $(\rho \ln \delta_D)(opt_{\alpha-2hop-DVB})$, where $opt_{\alpha-2hop-DVB})$ is the size of the optimal solution to α-2hop-DVB. In sum, Theorem 4 is proved. □

Next, we will prove the upper bound of our algorithm for the case $\alpha = 1$ in Theorem 5. We first give the definition of Set-Cover [26] before the proof.

Definition 4 (Set-Cover). *Given a collection $\mathcal{C}$ of subsets of a finite set X such that $\bigcup_{A \in \mathcal{C}} A = X$, find a minimum subcollection $\mathcal{A} \subseteq \mathcal{C}$ such that $\bigcup_{A \in \mathcal{A}} A = X$.*

Theorem 5. *Alg. 1 outputs the Sub_D with performance ratio $1 + \ln K + 2 \ln \delta_D$, where δ_D is the maximum* direction degree *of the input graph and K represents the number antennas deployed on each node, when $\alpha = 1$.*

Proof: During the computation of the new distributed algorithm, if a direction d is selected to join 1-MOC-DVB, we assign a weight $1/|pair(d)|$ to each pair (u, v) in $pair(d)$.

Suppose the minimum 1-2hop-DVB is constructed by $\{d_1^*, d_2^*, ..., d_k^*\}$, denoted as D_{Sub}^*. We estimate total weight collected at each direction d_i^*.

Initially, d_i^* has "f" value $f_0(d_i^*) = |pair(d_i^*)|$, having $f_0 < K * \delta_D^2$. After some directions are selected in the 1-MOC-DVB, $pair(d_i^*)$ is updated. Suppose for updated $pair(d_i^*)$, $f_1(d_i^*) = |pair(d_i^*)|$. $f_0(d_i^*) - f_1(d_i^*)$ is the number of pairs originally in $pair(d_i^*)$ and now are connected by those directions currently selected in the 1-MOC-DVB. Each such pair (u, v) has distance 2 such that u and v are adjacent to d_i^* and also adjacent to a new direction x in VB. By condition that x joins the 1-MOC-DVB, at least one of u and v sends flag to x. This means that before update, $f_0(d_i^*) = |pair(d_i^*)| \leq |pair(d)|$. Therefore, (u, v) received weight $1/f_0(d)$, where $1/f_0(d) \leq 1/f_0(d_i^*)$. All $f_0(d_i^*) - f_1(d_i^*)$ pairs receives weight at most $(f_0(d_i^*) - f_1(d_i^*))/f_0(d_i^*)$.

Similarly, we can prove that during the computation of this distributed algorithm, all pairs in $pair(d_i^*)$ received total weight at most

$$\sum_{i=0}^{i=k-1} \frac{f_i(d_i^*) - f_{i+1}(d_i^*)}{f_i(d_i^*)} \leq \sum_{i=1}^{i=f_0} \frac{1}{i} \leq 1 + \int_1^{f_0} (1/x)dx$$
$$= 1 + \ln K + 2 \ln \delta_D$$

where $f_k(d_i^*) = 0$ and δ_D is the maximum *direction degree* in the input graph.

Note that when one node is selected to D_{Sub}, the charged weight is 1. Thus, the total weight equals to the number of selected nodes, where D_{Sub} is the node set selected by Alg. 1. Therefore, we have $|D_{Sub}| \leq (1 + \ln K + 2\ln\delta_D) * opt$, where opt is the size of the minimum 1-MOC-DVB. □

6 Simulation

In this section, we conduct thorough simulation experiments to verify and analyze the performance of our algorithm. *M*aximum *R*outing *C*ost (MRC) denotes the maximum routing cost in one network between any pair of nodes while *A*verage *R*outing *C*ost (ARC) denotes the average routing cost among all pair of nodes in the network. Because of the use of uniform transmission range and uniform directional antennas, MRC and ARC can be evaluated by the routing length, that is the hop counts on paths. Our algorithm will be compared with other algorithms proposed for VB construction in terms of VB size, MRC, and ARC. Based on the definition of α-MOC-DVB, the source node will initiate a new message in all direction in our simulation. Our algorithm and VB will be compared with FKMS06 [27], ZJH06 [28], CDS-BD-D [5], ECC [4], and FlagContest [6].

6.1 Simulation Setup

In our simulation, nodes are randomly placed in a fixed area of $100m \times 100m$. In one network, the nodes have the same transmission radius varying among $15m$, $20m$, $25m$, and $30m$. We model the network allowing obstacles existence. However, for convenience, we conduct the simulation experiments without consideration of obstacles. The number of nodes n in each network is incremented from 10 to 100 by 10. The transmission range of each node is divided into K uniform sectors in a network and the degree of each sector is $360/K$. For one given set of n, K and transmission radius, 100 connected network instances are randomly generated. Experiment results are averaged among the 100 instances for each given set. The transmission model used in this paper is unicast.

6.2 Simulation Results

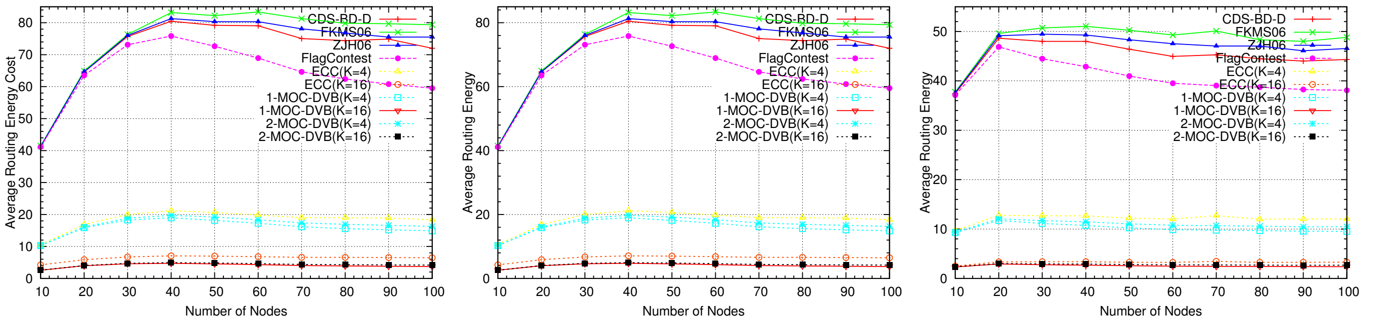
Fig. 6 and Fig. 7 show the comparisons in term of size among other VBs and the α-MOC-DVB selected by our distributed algorithm for the case $K = 4$ and $K = 16$ respectively, where $\alpha = 1$ or $\alpha = 2$. The transmission radius of the two figures vary among $15m$, $20m$, $25m$, and $30m$. From the two figures, we can tell that VB size becomes larger when the number of nodes increases for a given transmission range. More directions or nodes are selected to the VB because more nodes are needed to be dominated. The increase speed of VB size will slow down with the increase of node number because that when more nodes are placed in one fixed region, nodes' degrees will increase too and one node can dominate more nodes. The size of MOC-CDS is much larger than the size of other VBs because that MOC-CDS has shortest path restriction forcing more nodes added to the MOC-CDS and directional antennas are not used resulting energy waste in unwanted regions. 1-MOC-DVB has the shortest path constraint too. The size of α-MOC-DVB will decrease much, when we relax $\alpha = 1$ to $\alpha = 2$. The size of 2-MOC-DVB will decrease around 25%-60%, compared to 1-MOC-DVB. An α_1-MOC-DVB is also an α_2-MOC-DVB and α_1-MOC-DVB is more strict than α_2-MOC-DVB when $\alpha_1 < \alpha_2$. Thus, we can conclude that when we increase α, the VB size will decrease. Even though ECC selects a little bit fewer directions than 1-MOC-DVB, the tradeoff is that ECC needs to compute all paths with local information to select proper directions under some circumstances, which is not efficient.

The size of a VB in a network means how many nodes will be involved in broadcasting. The broadcasting costs will be reduced when the size of VB becomes smaller.

As shown in Fig. 8 and Fig. 9, ARC and MRC increase first and then decrease because the routing path length is more likely to increase when a new node is added in a connected network with small size of nodes. For example, a network with 1 node inside has ARC equal to 0. When a new node is connected to the network, both ARC and MRC will increase to 1. Hence, routing path length increases when n increases (n is relatively small). However, when n exceeds a certain value, newly added nodes are more likely to make distance between nodes smaller and the network more connected (considering physical space is fixed) which explains both MRC and ARC decrease. In addition, when transmission range increases, networks are more connected considering physical space is fixed. This explains the decrease of the routing cost in Fig. 8 and Fig. 9 compared to those left to it. The MRC and the ARC of our α-MOC-DVB is reduced around 90%, compared to those VB-based routing CDS-BD-D, FKMS06 and ZJH06 without consideration of routing cost and directional antennas. And our α-MOC-DVB is still around 67% better than MOC-CDS with shortest path constraint but not directional antenna. Meanwhile, we can also find that routing cost of 1-MOC-DVB is smaler than that of 2-MOC-DVB. Hence, we can conclude that there exists a tradeoff between the size of α-MOC-DVB and routing cost.

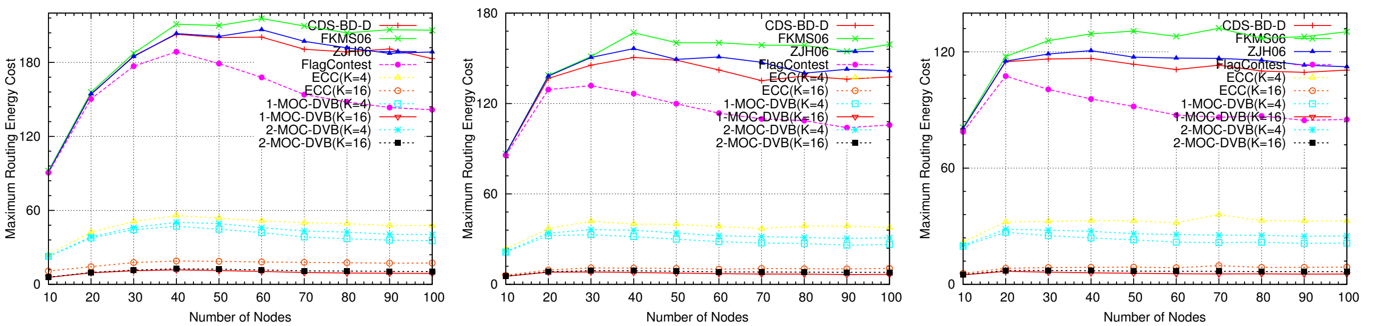
7 Conclusion

In this paper, we propose virtual backbone (VB) with guaranteed routing cost (α-MOC-DVB). It is proved that constructing a minimum α-MOC-DVB is NP-hard. There exists an unreachable lower bound of α-MOC-DVB. We propose a greedy distributed algorithm to construct α-MOC-DVB. When $\alpha = 1$, the performance ratio of our algorithm is $1 + \ln K + 2\ln\delta_D$ where δ_D is the maximum *direction degree* in the network and K represents the number of antennas deployed on each node in the network. The simulation results demonstrate the α-MOC-DVB outperforms many other VB works. Our future work includes further theoretical analysis of



(a) Transmission Range = 20 (b) Transmission Range = 25 (c) Transmission Range = 30

Fig. 8. Comparison of Average Routing Path Cost among CDS-BD-D, FKMS06, ZJH06, FlagContest and α-MOC-DVB in UDG Networks.



(a) Transmission Range = 20 (b) Transmission Range = 25 (c) Transmission Range = 30

Fig. 9. Comparison of Maximum Routing Cost among CDS-BD-D, FKMS06, ZJH06, FlagContest and α-MOC-DVB in UDG Networks.

our algorithm for the case $\alpha > 1$ and propose algorithm with better performance ratio.

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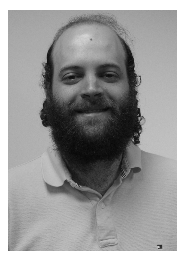

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